Optimism and overconfidence

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Motivation

Wrong beliefs are pervasive: vast lit in psychology & economics.

Two distinct (but oft-conflated) senses of 'wrong':

- overconfidence: overestimate ability to influence outcomes
- optimism: overestimate chances of 'good' outcomes

Seek to define, distinguish & characterise these.

- behavioural, model-free definitions
- characterisation in the canonical model of an effort-influencing agent: the moral-hazard (MH) model

Prior question: how does the MH model relate to behaviour?

- how does it restrict choice behaviour? (testable axioms)
- can its parameters be recovered from choice behaviour?

This paper

Data: choice between contracts.

Proposition 1: charac'n of MH model's empirical content
(six axioms exhaust its testable implications).

Proposition 2: charac'n of extent to which MH model's
parameters can be recovered from data.

Definitions {
of 'more confident than'
of 'more optimistic than'

Proposition 3: charac'n of parameter shifts in MH model
that increase confidence.

Proposition 4: charac'n of parameter shifts in MH model
that increase optimism.

Method: establish link with 'variational' model, borrow results.

Environment

Convex set $\Pi \subseteq \mathbf{R}$ of possible levels of remuneration.

Finite set S of possible realisations of output.

A contract is a map $w : S \to \Delta(\Pi)$.

($\Delta(\Pi)$ = set of all finite-support probabilities on Π.)

Write W for the set of all contracts.

Interpretation:

– agent's pay $\pi \in \Pi$ can be contingent on output $s \in S$

– 'output' can be any contractible signal/outcome

– pay can be random conditional on output (for simplicity)

Definitions and conventions

Elements of $\Delta(\Pi)$ are called random remunerations.

A contract w is constant iff $w(s) = w(s')$ for all $s \in S$.

Convention: identify each constant contract $(\in W)$
with the random remuneration $(\in \Delta(\Pi))$
at which it is constant.

Convention: extend any (utility) function $u : \Pi \to \mathbf{R}$
to an (expected-utility) function $\Delta(\Pi) \to \mathbf{R}$
via $u(x) \coloneqq \int_\Pi u(\pi) x(\mathrm{d}\pi) \quad \forall x \in \Delta(\Pi)$.

Throughout, fix arbitrary $\pi_0 < \pi_1$ in Π.
Call a (utility) function $u : \Pi \to \mathbf{R}$ such that $u(\pi_0) \neq u(\pi_1)$
normalised iff $\{u(\pi_0), u(\pi_1)\} = \{0, 1\}$.

All sets $\subseteq \mathbf{R}^n$ (incl. $\Delta(S)$) have the Borel σ-algebra.

The standard moral-hazard model

Standard MH model has four parameters:

- a compact convex (effort) set $E \subseteq \mathbf{R}^n$
- a grounded[1] and lsc[2] (cost) function $C : E \to \mathbf{R}_+$
- a continuous (belief) map $e \mapsto P_e$ that carries E into $\Delta(S)$
- a strictly $\nearrow$ & normalised (utility) function $u : \Pi \to \mathbf{R}$

Under contract $w \in W$, agent chooses effort $e \in E$
to max expected utility from remuneration net of effort cost:

$$\sup_{e \in E} \left[-C(e) + \sum_{s \in S} u(w(s)) P_e(s) \right]$$

[1] Viz. $\inf_{e \in E} C(e) = 0$.

[2] That's 'lower semi-continuous'.

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Under contract $w \in W$, agent chooses effort dist'n $\mu \in \Delta(E)$ to max expected utility from remuneration net of effort cost:

$$\sup_{\mu \in \Delta(E)} \int_E \left[-C(e) + \sum_{s \in S} u(w(s)) P_e(s) \right] \mu(\mathrm{d}e)$$

(Randomising may be strictly optimal.)

[1]Viz. $\inf_{e \in E} C(e) = 0$.

[2]That's 'lower semi-continuous'.

Data: preferences over contracts

Agent's preference over contracts: binary relation $\succeq$ on W.

This is (in principle) data: choice between contracts.

$\succeq$ is a MH preference iff there are parameters $(E, C, e \mapsto P_e, u)$
(which satisfy the required properties, see last slide)
such that $w \succeq w'$ iff

$$\sup_{\mu \in \Delta(E)} \int_E \left[-C(e) + \sum_{s \in S} u(w(s)) P_e(s) \right] \mu(\mathrm{d}e)$$
$$\geq \sup_{\mu \in \Delta(E)} \int_E \left[-C(e) + \sum_{s \in S} u(w'(s)) P_e(s) \right] \mu(\mathrm{d}e).$$

Assumption: no data besides $\succeq$. (Effort is unobservable.)

A parsimonious moral-hazard model

Can break agent's problem into:

(1) choose which output distribution $p \in \Delta(S)$ to produce

(2) find least-cost way of producing p:
search among all $\mu \in \Delta(E)$ such that $\int_E P_e \mu(\mathrm{d}e) = p$.

Solving (2) yields least cost of producing each $p \in \Delta(S)$:

$$c(p) := \inf_{\substack{\mu \in \Delta(E): \\ \int_E P_e \mu(\mathrm{d}e) = p}} \int_E C(e)\mu(\mathrm{d}e),$$

where $c(p) = \infty$ if the constraint set is empty.

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Solving (1) yields value of contract $w \in W$:

$$\sup_{\mu \in \Delta(E)} \int_E \left[-C(e) + \sum_{s \in S} u(w(s)) P_e(s) \right] \mu(\mathrm{d}e)$$

$$= \sup_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s)) p(s) \right].$$

A parsimonious moral-hazard model

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$$\text{Value of } w = \sup_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s)) p(s) \right]$$

Only parameters that matter: (c, u).

Henceforth use the parsimonious parametrisation: just (c, u).

The power of parsimony

Parsimonious parametrisation (c, u) is very well-behaved:

Lemma: relation $\succeq$ is a MH preference iff there is

– a grounded, <u>convex and lsc</u> function $c : \Delta(S) \to [0, \infty]$ and

– a strictly $\nearrow$ and normalised (utility) function $u : \Pi \to \mathbf{R}$

such that $w \succeq w'$ iff

$$\max_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s))p(s) \right] \geq \max_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w'(s))p(s) \right].$$

Call (c, u) a <u>parsimonious (MH) representation</u> of $\succeq$.

The power of parsimony: necessity

Already showed value of $w = \sup_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s)) p(s) \right]$

where $c(q) \coloneqq \inf_{\substack{\mu \in \Delta(E): \\ \int_E P_e \mu(\mathrm{d}e) = q}} \int_E C(e) \mu(\mathrm{d}e)$ for each $q \in \Delta(S)$.

<u>c lsc:</u> since C lsc and $e \mapsto P_e$ continuous.

<u>c convex:</u> by construction.

Given $q, q' \in \Delta(S)$, let $\mu, \mu' \in \Delta(E)$ be least-cost effort dist'ns.

Effort dist'n $\alpha\mu + (1-\alpha)\mu'$ $\begin{cases} \text{produces} & \alpha q + (1-\alpha) q' \\ \text{costs} & \alpha c(q) + (1-\alpha) c(q'). \end{cases}$

So $c(\alpha q + (1-\alpha) q') \leq \alpha c(q) + (1-\alpha) c(q')$.

The power of parsimony: necessity

Already showed value of $w = \max_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s)) p(s) \right]$

where $c(q) := \inf_{\substack{\mu \in \Delta(E): \\ \int_E P_e \mu(\mathrm{d}e) = q}} \int_E C(e) \mu(\mathrm{d}e)$ for each $q \in \Delta(S)$.

$\underline{c \text{ lsc:}}$ since C lsc and $e \mapsto P_e$ continuous.

$\Longrightarrow$ can replace 'sup' with 'max'.

$\underline{c \text{ convex:}}$ by construction.

Given $q, q' \in \Delta(S)$, let $\mu, \mu' \in \Delta(E)$ be least-cost effort dist'ns.

Effort dist'n $\alpha\mu + (1-\alpha)\mu'$ $\begin{cases} \text{produces} & \alpha q + (1-\alpha) q' \\ \text{costs} & \alpha c(q) + (1-\alpha) c(q'). \end{cases}$

So $c(\alpha q + (1-\alpha) q') \leq \alpha c(q) + (1-\alpha) c(q')$.

The power of parsimony: sufficiency

Suppose $\succeq$ admits parsimonious MH representation (c, u).

– let $E := \Delta(S)$

– define $C : E \to \mathbf{R}_+$ by $C \equiv c$

– let $e \mapsto P_e$ be the identity $(P_p = p$ for each $p \in \Delta(S))$

The MH model $(E, C, e \mapsto P_e, u)$ represents $\succeq$: $\forall w \in W$,

$$\max_{p \in \Delta(S)} \left[-c(p) + \sum_{s \in S} u(w(s)) p(s) \right]$$

$$= \sup_{\mu \in \Delta(E)} \int_E \left[-C(e) + \sum_{s \in S} u(w(s)) P_e(s) \right] \mu(\mathrm{d}e).$$

So $\succeq$ is a MH preference. **QED**

Four testable implications of the MH model

Axiom 1: $\succeq$ is complete and transitive.

Axiom 2: For any prizes $\pi, \pi' \in \Pi$, $\pi > \pi'$ implies $\pi \succ \pi'$.

Axiom 3: If two contracts $w, w' \in W$ satisfy $w(s) \succeq w'(s)$ for every output level $s \in S$, then $w \succeq w'$.

Axiom 4: For any contracts $w, w', w'' \in W$, the sets $\{\alpha \in [0,1] : \alpha w + (1-\alpha)w' \succeq w''\}$ and $\{\alpha \in [0,1] : w'' \succeq \alpha w + (1-\alpha)w'\}$ are closed.

More testable implications of the MH model

Quasiconvexity: For any contracts $w, w' \in W$
such that $w \succeq w' \succeq w$,
$w \succeq \alpha w + (1-\alpha)w'$ for all $\alpha \in (0,1)$.

Interpretation: aversion to 'mixing' contracts.

MMR Independence: For any $w, w' \in W$ and $\alpha \in (0,1)$,

$$\alpha w + (1-\alpha)y \succeq \alpha w' + (1-\alpha)y \quad \text{for some } y \in \Delta(\Pi)$$
$$\Longrightarrow \quad \alpha w + (1-\alpha)y' \succeq \alpha w' + (1-\alpha)y' \quad \text{for any } y' \in \Delta(\Pi).$$

One interpretation: absence of income effects.

Empirical content of the MH model

Proposition 1: A relation $\succeq$ on W is a MH preference iff it satisfies Axioms 1–4, MMR Independence, and Quasiconvexity.

Proof: borrow from Maccheroni–Marinacci–Rustichini's (2006) axiomatisation of 'variational' preferences. Similar to MH, except malevolent Nature chooses effort (and bears the cost).

Behavioural difference: Quasiconvexity vs. Quasiconcavity.

Identification of the MH model

$\succeq$ unbounded $\simeq$ utility function unbounded above and below.[3]

Proposition 2: Each unbounded MH preference admits exactly one parsimonious representation.

Proof: borrow from MMR again.

Good news: parsimonious MH model fully identified.

Bad news: standard MH model not identified.
Can't recover $(E, C, e \mapsto P_e)$.

More data may or may not help:
- not helpful: observing the produced output dist'n
- helpful: observing chosen effort

[3] Real definition: there are $x \succ y$ in $\Delta(\Pi)$ such that for any $\alpha \in (0,1)$, we may find $z, z' \in \Delta(\Pi)$ that satisfy $y \succ \alpha z + (1-\alpha)x$ and $\alpha z' + (1-\alpha)y \succ x$.

Relative confidence

Confident agent: one who believes she can significantly influence the distribution of output.

In terms of choice: greater appetite for non-constant contracts.

Definition: $\succeq$ is <u>more confident than</u> $\succeq'$ iff for any $w \in W$ and $x \in \Delta(\Pi)$,

$$w \succeq' (\succ')\, x \quad \Longrightarrow \quad w \succeq (\succ)\, x.$$

Relative confidence in the MH model

Proposition 3: Let $\succeq$ and $\succeq'$ be MH preferences,
with parsimonious rep'ns (c, u) and (c', u').
Then $\succeq$ is more confident than $\succeq'$
iff $u = u'$ and $c \leq c'$.

$\iff$ (c, u) is more confident than (c', u') iff $u = u'$ and

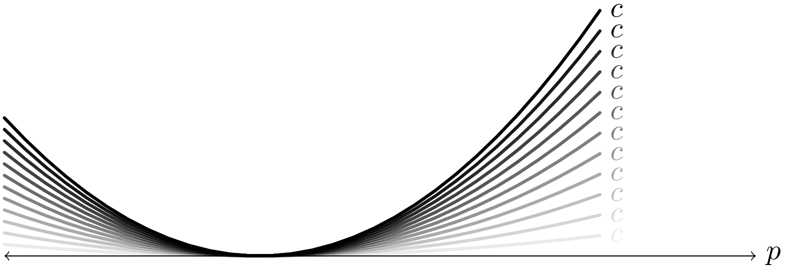
$\{p \in \Delta(S) : c(p) \leq k\} \supseteq \{p \in \Delta(S) : c'(p) \leq k\}$ for every $k \geq 0$.

<u>Proof:</u> Borrow from MMR again!

Relative confidence in the MH model: picture

Two output levels: $S = \{\text{failure}, \text{success}\}$.

Can view each $p \in \Delta(S)$ as one-dimensional: $p \equiv \Pr(\text{success})$.



confidence ↑ $\iff$ $\boldsymbol{c}$ **↓**

In the MH model, confidence is about vertical shifts of c.

Relative optimism

Henceforth $S = \{s_1, s_2, \ldots, s_{|S|}\}$, where $s_1 < s_2 < \cdots < s_{|S|}$.

Optimistic agent: one who expects output to be high.

In terms of choice: greater appetite for steeply $\nearrow$ contracts.

Appropriate sense of 'steeply' adjusts for risk attitude:
steepness of $u \circ w$ (in utils), not of w (in dollars).

Definition: $\succeq$ is <u>more optimistic than</u> $\succeq'$ iff
they have the same (EU) risk attitude u, and
for any $w, w' \in W$ such that $u \circ w - u \circ w'$ is $\nearrow$,
$w \succeq' (\succ') \, w' \implies w \succeq (\succ) \, w'$.

Up-shiftedness

Let $c, c' : \Delta(S) \to [0, \infty]$ be grounded, convex and lsc.

c is up-shifted from c' iff $\forall p, p' \in \Delta(S), \quad \exists q, q' \in \Delta(S) \quad$ s.t.

- p FOSD q'
- q FOSD p'
- $\frac{1}{2}p + \frac{1}{2}p' = \frac{1}{2}q + \frac{1}{2}q'$
- $c(q) + c'(q') \leq c(p) + c'(p')$.

Idea: FOSD-higher output dist'ns are relatively cheaper under c than under c'.

Concretely (Dziewulski–Quah, 2024): c is up-shifted from c' iff for every contract $w \in W$ and strictly $\nearrow$ utility $u : \Pi \to \mathbf{R}$, optimal 'effort' $p \in \Delta(S)$ is FOSD-higher in MH model (c, u) than in MH model (c', u).

Up-shiftedness: picture

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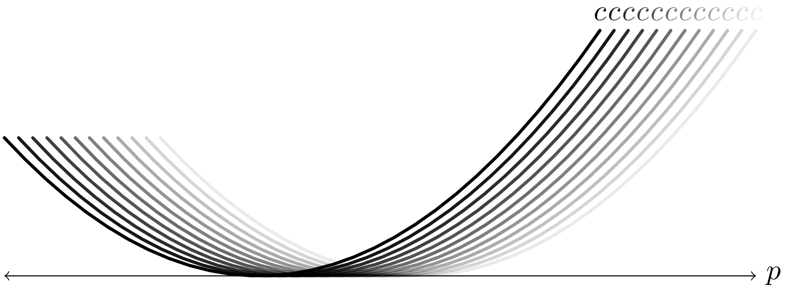
Can view each $p \in \Delta(S)$ as one-dimensional: $p \equiv \Pr(\text{success})$.



up-shift $\iff$ $\boldsymbol{c} \rightarrow$

Up-shifting is about <u>horizontal</u> shifts.

Up-shiftedness: a sufficient condition

Let
$$L_k := \{p \in \Delta(S) : c(p) \leq k\}$$
$$L'_k := \{p \in \Delta(S) : c'(p) \leq k\}.$$

Obs'n: Let $c, c' : \Delta(S) \to [0, \infty]$ be grounded, convex and lsc. If c is up-shifted from c', then for every $k \geq 0$,

for each $p \in L_k$, p FOSD p' for some $p' \in L'_k$, and
for each $p' \in L'_k$, p FOSD p' for some $p \in L_k$.

Intuitively: the set L_k is 'FOSD-higher' than the set L'_k.

<ins>Proof of the first half:</ins> fix $k \geq 0$ and $p \in L_k$.
c' grounded and lsc $\implies$ $\exists p' \in \Delta(S)$ such that $c'(p') = 0$.
By up-shiftedness, $\exists q, q' \in \Delta(S)$ such that p FOSD q' and
$c'(q') \leq c(q) + c'(q') \leq c(p) + c'(p') \leq k \implies q' \in L'_k$. **QED**

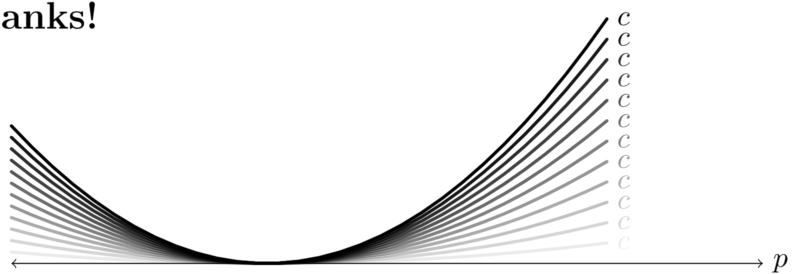
Relative optimism in the MH model

Proposition 4: Let $\succeq$ and $\succeq'$ be MH preferences,
with parsimonious rep'ns (c, u) and (c', u').
Then $\succeq$ is more optimistic than $\succeq'$
iff $u = u'$ and c is up-shifted from c'.

$\Longrightarrow$ in MH model, optimism shifts are horizontal shifts of c.

Proof: Borrow from Dziewulski and Quah (2024).

Thanks!



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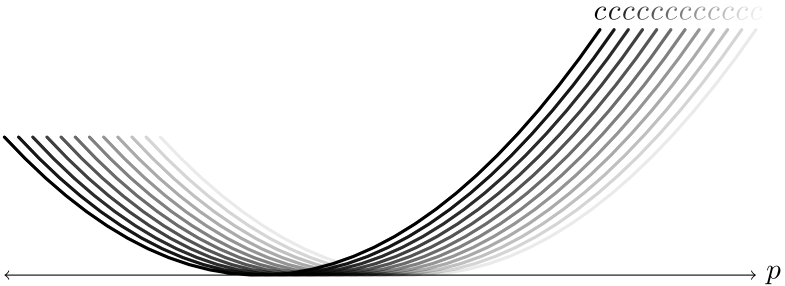

optimism ↑ $\iff$ c →

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