

Comparative statics with adjustment costs and the le Chatelier principle

Eddie Dekel
Northwestern
& Tel Aviv

John Quah
NUS

Ludvig Sinander
Oxford

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Motivation

Comparative statics: under what circumstances
does a parameter shift
'increase' optima / equilibria?

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Adjustment costs: key feature of many economic models, e.g.

- capital investment (e.g. Jorgenson, 1963; Hayashi, 1982; Cooper & Haltiwanger, 2006)
- sticky prices (e.g. Mankiw, 1985; Caplin & Spulber, 1987; Golosov & Lucas, 2007; Midrigan, 2011)
- trade in illiquid assets (e.g. Kyle, 1985; Back, 1992)
- wealthy hand-to-mouth (e.g. Kaplan & Violante, 2014; Berger & Vavra, 2015; Chetty & Szeidl, 2016)
- labour supply (e.g. Chetty, Friedman, Olsen & Pistaferri, 2011; Chetty, 2012)
- labour demand (e.g. Hamermesh, 1988; Bentolila & Bertola, 1990)

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'increase' optima / equilibria?

Adjustment costs: key feature of many economic models.

This paper: comparative statics for adjustment-cost models.

Example: sticky-price models

Central plank of new Keynesian macro models: sticky prices.

Most important micro-foundation: adjustment ('menu') costs.
(e.g. Mankiw, 1985; Golosov & Lucas, 2007; Midrigan, 2011)

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Simplest model: monopolist with constant marg. cost $c \geq 0$,
decr. demand curve $D(\cdot, \eta)$,
parameter η shifts |elasticity|.
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Under what ass'ns on demand $D(\cdot, \eta)$ & adj. cost $C(\cdot)$ do

supply shocks ($c \nearrow$) \implies inflation?

demand-elasticity shocks ($\eta \searrow$) \implies inflation?

Overview

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Basic insight (Th'm 1): need only very weak
assumptions on cost.

Consequence (Th'm 2): new, greatly generalised
le Chatelier principle.

Consequence (Th'ms 3–6): results extend to
infinite-horizon model.

Applications: factor demand, pricing, investment,
labour supply, saving.

Setting

Action $x \in L$, $L \subseteq \mathbf{R}^n$ (L a sublattice)

Objective $F(x, \theta)$ depends on parameter θ
($\in \Theta$, a partially ordered set)

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Adjusting by $\epsilon = x - \underline{x}$ costs $C(\epsilon) \geq 0$

Agent maximises $G(x, \bar{\theta}) := F(x, \bar{\theta}) - C(x - \underline{x})$.

Cost assumptions

Cost $C : \Delta L \rightarrow [0, \infty]$ where $\Delta L := \{x - y : x, y \in L\} \subseteq \mathbf{R}^n$.

Assume little about C : $C(0) < \infty$ and

- for first result : C minimally monotone
- for other results: C monotone

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Allows

- non-convex costs (even non-quasiconvex)
- some adjustments infeasible: $C(\epsilon) = \infty$
- non-separability between dimensions (as in Midrigan (2011))

Minimal monotonicity

C is minimally monotone iff $C(\epsilon \wedge 0) \leq C(\epsilon) \geq C(\epsilon \vee 0) \quad \forall \epsilon.$

(‘ \wedge ’ is entry-wise min; ‘ \vee ’ is entry-wise max.)

Interpretation: cancelling all upward adjustments lowers cost;
likewise for downward adjustments.

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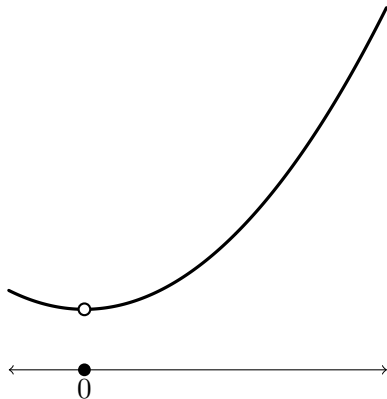
Minimal monotonicity: example

1D action $L \subseteq \mathbf{R}$

fixed cost $k > 0$

variable cost $a\epsilon^2$ ($a > 0$)

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } \epsilon \neq 0 \end{cases}$$



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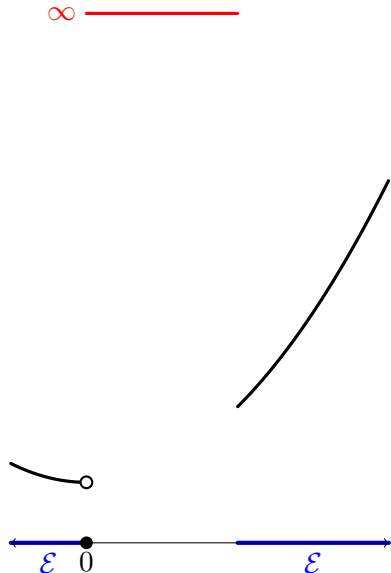
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As in Field–Pande–Papp–Rigol 2013,
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Review: costless adjustment

Basic result (see Milgrom & Shannon, 1994): if $F(x, \theta)$ exhibits

(1) complementarity btw. action x & parameter θ

(2) complementarity btw. action dimensions $x = (x_1, \dots, x_n)$

then $\bar{x} \geq \underline{x}$ for some $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$ provided $\arg \max$ is not empty.

Here

(1) means single-crossing differences in (x, θ) : for any $x' > x$,

$$F(x', \theta) \geq (>) F(x, \theta) \implies F(x', \theta') \geq (>) F(x, \theta') \quad \text{if } \theta < \theta'$$

(2) means quasi-supermodularity in x : for any $x, x' \in L$,

$$F(x, \theta) \geq (>) F(x \wedge x', \theta) \implies F(x \vee x', \theta) \geq (>) F(x', \theta) \quad \forall \theta$$

Review: ordinal vs. cardinal complementarity

SCD & QSM are

- ordinal: preserved by \nearrow transformations
- not inherited by sums: sum of SCD (QSM) functions
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- ordinal: preserved by \nearrow transformations
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Cardinal sufficient conditions

that are (not ordinal, but) inherited by sums:

incr. differences in (x, θ) : marginal return $F(x', \theta) - F(x, \theta)$
 \nearrow in θ (for $x' > x$)

supermodularity in x : marginal return
 $F((x'_i, x_j, x_{-ij}), \theta) - F((x_i, x_j, x_{-ij}), \theta)$
 \nearrow in x_j (for $x'_i > x_i, i \neq j$)

Comparative statics with costly adjustment

Recall: agent maximises $G(x, \bar{\theta}) := F(x, \bar{\theta}) - C(x - \underline{x})$.

Theorem 1: if $F(x, \theta)$ exhibits

- (1) complementarity btw. action x & parameter θ
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and cost C is minimally monotone,

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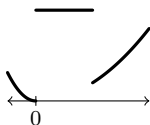
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Costs need not even be single-dipped! E.g. $C =$



Only ordinal complementarity on F . (Not inherited by G !)

Proof of Theorem 1

Fix $x' \in \arg \max_{x \in L} G(x, \bar{\theta})$.

Clearly $\underline{x} \vee x' \geq \underline{x}$. Will show $\underline{x} \vee x' \in \arg \max_{x \in L} G(x, \bar{\theta})$.

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Standard steps:

$$\begin{aligned} F(\underline{x}, \underline{\theta}) &\geq F(\underline{x} \wedge x', \underline{\theta}) && \text{by def'n of } \underline{x} \\ \implies F(\underline{x} \vee x', \underline{\theta}) &\geq F(x', \underline{\theta}) && \text{by QSM} \\ \implies F(\underline{x} \vee x', \bar{\theta}) &\geq F(x', \bar{\theta}) && \text{by SCD.} \end{aligned}$$

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New step:

$$\begin{aligned} &C(\underline{x} \vee x' - \underline{x}) \\ &= C((x' - \underline{x}) \vee 0) \\ &\leq C(x' - \underline{x}) && \text{by minimal monotonicity.} \end{aligned}$$

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So $G(\underline{x} \vee x', \bar{\theta}) = F(\underline{x} \vee x', \bar{\theta}) - C(\underline{x} \vee x' - \underline{x})$
 $\geq F(x', \bar{\theta}) - C(x' - \underline{x}) = G(x', \bar{\theta}).$ **QED**

Monotonicity

Cost C is monotone iff for each adj. vector ϵ & each i ,

$$C(\epsilon_1, \dots, \epsilon_{i-1}, \epsilon'_i, \epsilon_{i+1}, \dots, \epsilon_n) \leq C(\epsilon) \quad \text{whenever } 0 \leq \epsilon'_i \leq \epsilon_i \\ \text{or } 0 \geq \epsilon'_i \geq \epsilon_i.$$

Interpretation: adjusting less lowers cost.

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For $L \subseteq \mathbf{R}$, C monotone
 $\iff C$ single-dipped & minimised at 0.
(i.e. \searrow on $(-\infty, 0]$, \nearrow on $[0, \infty)$)

For additively separable $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$,

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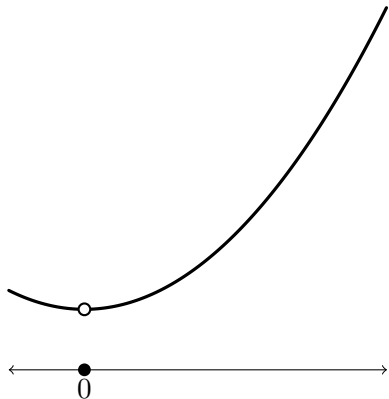
Example of monotonicity

1D action $L \subseteq \mathbf{R}$

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variable cost $a\epsilon^2$ ($a > 0$)

$$C(\epsilon) = \begin{cases} 0 & \text{for } \epsilon = 0 \\ k + a\epsilon^2 & \text{for } \epsilon \neq 0 \end{cases}$$



Example of monotonicity violation

1D action $L \subseteq \mathbf{R}$

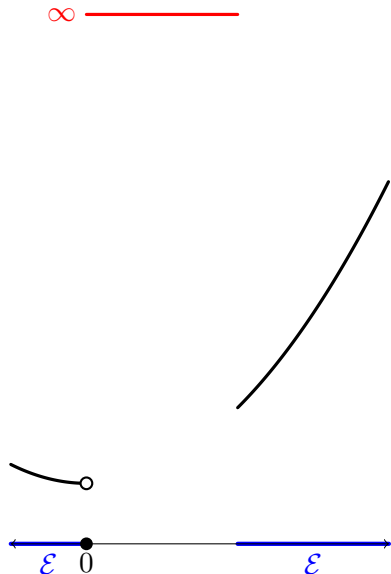
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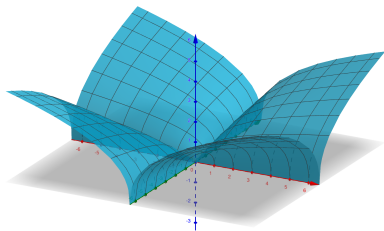
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each C_i single-dipped & minimised at 0.
- Euclidean: $C(\epsilon) = \sqrt{\sum_{i=1}^n \epsilon_i^2}$.
- Cobb–Douglas: $C(\epsilon) = \prod_{i=1}^n |\epsilon_i|^{a_i}$
where $a_1, \dots, a_n > 0$.

(not quasiconvex)



The le Chatelier principle

Le Chatelier principle: $|\text{LR elasticity}| \geq |\text{SR elasticity}|$.

Usual story: only some dimensions of x adjustable in SR.

¹Such \bar{x} exists by the ‘basic result’ (sl. 9), provided argmax nonempty.

The le Chatelier principle

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Usual story: only some dimensions of x adjustable in SR.

Formalisation (Milgrom & Roberts, 1996): suppose $F(x, \theta)$ exhibits

(1) complementarity btw. action x & parameter θ

(2) complementarity btw. action dimensions $x = (x_1, \dots, x_n)$.

Let $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$ satisfy $\bar{x} \geq \underline{x}$.¹ (LR optimum.)

Then $\bar{x} \geq \hat{x} \geq \underline{x}$ for some SR-optimal \hat{x} .

¹Such \bar{x} exists by the ‘basic result’ (sl. 9), provided argmax nonempty.

General le Chatelier principle

Different story: SR adjustment is costly.

Nests usual story as (very) special case: $C(\epsilon) = \sum_{i=1}^n C_i(\epsilon_i)$,

– some dimensions have $C_i \equiv 0$

– other dimensions have $C_i(\epsilon_i) = \infty$ for every $\epsilon_i \neq 0$.

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Theorem 2: suppose $F(x, \theta)$ exhibits

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and cost C is monotone.

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Then $\bar{x} \geq \hat{x} \geq \underline{x}$ for some $\hat{x} \in \arg \max_{x \in L} G(x, \bar{\theta})$ provided $\arg \max$ is not empty.

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Proof of Theorem 2

By Theorem 1, we may choose $x' \geq \underline{x}$ in $\arg \max_{x \in L} G(x, \bar{\theta})$.

Clearly $\bar{x} \geq \bar{x} \wedge x' \geq \underline{x}$. We show $\bar{x} \wedge x' \in \arg \max_{x \in L} G(x, \bar{\theta})$.

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New step: $x' \geq \bar{x} \wedge x' \geq \underline{x} \implies (x' - \underline{x}) \geq (\bar{x} \wedge x' - \underline{x}) \geq 0$
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 $= G(\bar{x} \wedge x', \bar{\theta}).$

QED

Why not minimal monotonicity?

Minimal monotonicity is not enough for le Chatelier. Example:

$$L = \mathbf{R} \quad \begin{aligned} F(x, \underline{\theta}) &= -x^2 \\ F(x, \bar{\theta}) &= -(x - 2)^2 \end{aligned} \quad C(\epsilon) = \begin{cases} \infty & \text{if } 0 < \epsilon < 3 \\ 0 & \text{otherwise.} \end{cases}$$



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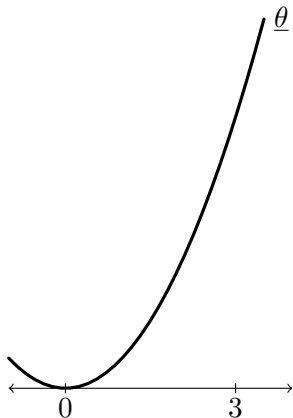
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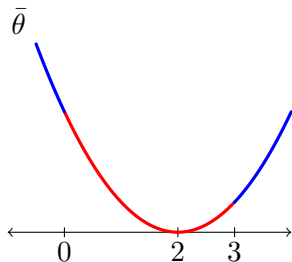
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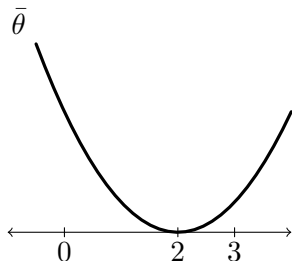
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LR: $\bar{x} = 2 \in \arg \min_{x \in \mathbf{R}} (x - 2)^2$



Application to factor demand

Inputs (k, ℓ) , real input prices (r, w) , production function f

Monotone adjustment cost

Profit $F(k, \ell, -w) = f(k, \ell) - rk - w\ell$

- supermodular in (k, ℓ) : if f supermodular (complements)
- incr. diff. in $((k, \ell), -w)$: $\nabla_{(k, \ell)} F = \begin{pmatrix} f_k - r \\ f_\ell - w \end{pmatrix} \nearrow$ in $-w$

Application to factor demand

Inputs (k, ℓ) , real input prices (r, w) , production function f

Monotone adjustment cost

Profit $F(k, \ell, -w) = f(k, \ell) - rk - w\ell$

– supermodular in (k, ℓ) : if f supermodular (complements)

– incr. diff. in $((k, \ell), -w)$: $\nabla_{(k, \ell)} F = \begin{pmatrix} f_k - r \\ f_\ell - w \end{pmatrix} \nearrow$ in $-w$

As $w \searrow$, both factor demands \nearrow (Theorem 1)

In LR, both factor demands further \nearrow (Theorem 2)

If instead f submodular (substitutes), ℓ demand \nearrow but
 k demand \searrow .

Application to pricing

Menu-cost model: monopolist with constant marg. cost $c \geq 0$,
decr. demand curve $D(\cdot, \eta)$,
parameter η shifts |elasticity|.
Adjusting price by ϵ costs $C(\epsilon) \geq 0$.

Application to pricing

Menu-cost model: monopolist with constant marg. cost $c \geq 0$,
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Adjusting price by ϵ costs $C(\epsilon) \geq 0$.

Profit $F(p, (c, -\eta)) = (p - c)D(p, \eta)$

- F QSM in p : automatic since p one-dimensional ($\in \mathbf{R}$)
- $\ln F$ has incr. differences in $(p, (c, -\eta))$:

$$\frac{d}{dp} \ln F = \frac{1}{p - c} + \frac{D'(p, \eta)}{D(p, \eta)} = \frac{1}{p - c} - \frac{|\text{elasticity}(p, \eta)|}{p}$$

\nearrow in c & $-\eta$

Dynamic adjustment

Agent chooses $x_t \in L$ in each period $t \in \mathbf{N}$

Period- t payoff: $F(x_t, \theta_t) - C_t(x_t - x_{t-1})$

Taken as given:

- initial choice $x_0 = \underline{x} \in \arg \max_{x \in L} F(x, \underline{\theta})$
- parameter path $(\theta_t)_{t=1}^{\infty}$
- cost path $(C_t)_{t=1}^{\infty}$

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- cost path $(C_t)_{t=1}^{\infty}$

Agent forward-looking, discount rate $\delta \in (0, 1)$

Chooses path $(x_t)_{t=1}^{\infty}$ to max $\sum_{t=1}^{\infty} \delta^t [F(x_t, \theta_t) - C_t(x_t - x_{t-1})]$.

Dynamic le Chatelier principle

Theorem 3: suppose $F(x, \theta)$ exhibits

- (1) complementarity btw. action x & parameter θ
 - (2) complementarity btw. action dimensions $x = (x_1, \dots, x_n)$,
- and that each cost C_t is monotone.

Fix $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$ such that $\bar{x} \geq \underline{x}$.³

If $\underline{\theta} \leq \theta_t \leq \bar{\theta} \quad \forall t$, then $\underline{x} \leq x_t \leq \bar{x} \quad \forall t$

for some solution $(x_t)_{t=1}^{\infty}$, provided a solution exists.

Proof: straightforward extension of Theorem 1+2 logic.

³Such \bar{x} exists by the ‘basic result’ (sl. 9), provided argmax nonempty.

Strong dynamic le Chatelier principle

Theorem 4: suppose $F(x, \theta)$ exhibits

- (1) complementarity btw. action x & parameter θ
- (2) cardinal complementarity btw. dimensions $x = (x_1, \dots, x_n)$
- (3) boundedness in x on each compact set $\subseteq L$, for each θ ,

and that $C_t = C \ \forall t$ for C monotone & additively separable.

Fix $\bar{x} \in \arg \max_{x \in L} F(x, \bar{\theta})$ such that $\bar{x} \geq \underline{x}$.⁴

If $\theta_t = \bar{\theta} \ \forall t$, then $\underline{x} \leq x_t \leq x_{t+1} \leq \bar{x} \ \forall t$

for some solution $(x_t)_{t=1}^\infty$, provided a solution exists.

⁴Such \bar{x} exists by the ‘basic result’ (sl. 9), provided argmax nonempty.

Sketch proof of Theorem 4

Can ‘monotonise’ any $(x_t)_{t=1}^{\infty}$ by replacing t^{th} entry
with cumulative max $x_1 \vee x_2 \vee \cdots \vee x_{t-1} \vee x_t$.

Claim: monotonicisation preserves optimality.

(Suffices since can then monotonicise solution from Theorem 3.)

Sketch proof of Theorem 4

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with cumulative max $x_1 \vee x_2 \vee \cdots \vee x_{t-1} \vee x_t$.

Claim: monotonicisation preserves optimality.

(Suffices since can then monotonicise solution from Theorem 3.)

By boundedness & limit argument, suffices to show that

if $(x_t)_{t=1}^{\infty}$ optimal & $x_1 \leq x_2 \leq \cdots \leq x_{k-1} \leq x_k$,

remains optimal if replace t^{th} entry by $x_{t-1} \vee x_t \quad \forall t \geq k + 1$.

Proved using supermodularity of $F(\cdot, \bar{\theta})$,
+ monotonicity & additive separability of C .

(argument: slide 28)

Application to pricing, continued

Assume cost C time-invariant,
demand $D(\cdot, \eta)$ upper semi-continuous.

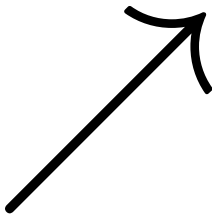
Profit $F(p, (c, -\eta)) = (p - c)D(p, \eta)$

- $F(p, (c, -\eta))$ SM in p & C additively separable:
automatic since p one-dimensional ($\in \mathbf{R}$)
- $F(p, (c, -\eta))$ bounded in p on each compact set $\subseteq \mathbf{R}_+$

Theorem 4:

- supply shock ($c \nearrow$) \implies inflation at every horizon
- demand shock s.t. $\eta \searrow$ \implies inflation at every horizon

Thanks!



Sketch proof of Theorem 4: main step

$(x_t)_{t=1}^{\infty}$ optimal \implies better than $(x_t \wedge x_{t+1})_{t=1}^{\infty}$:

$$\begin{aligned} & \sum_{t=k}^{\infty} \delta^{t-k} \left[F(x_t, \bar{\theta}) - F(x_t \wedge x_{t+1}, \bar{\theta}) \right] \\ & - \sum_{t=k}^{\infty} \delta^{t-k} \left[C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t) \right] \geq 0. \end{aligned}$$

Sketch proof of Theorem 4: main step

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$F(\cdot, \bar{\theta})$ supermodular: $F(x_t \vee x_{t+1}, \bar{\theta}) - F(x_{t+1}, \bar{\theta})$
 $\geq F(x_t, \bar{\theta}) - F(x_t \wedge x_{t+1}, \bar{\theta}) \quad \forall t \geq k$

C monotone & additively separable: (argument omitted)

$$\begin{aligned} & C(x_t \vee x_{t+1} - x_{t-1} \vee x_t) - C(x_{t+1} - x_t) \\ & \leq C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t) \quad \forall t \geq k \end{aligned}$$

Sketch proof of Theorem 4: main step

$(x_t)_{t=1}^{\infty}$ optimal \implies better than $(x_t \wedge x_{t+1})_{t=1}^{\infty}$:

$$\begin{aligned} & \sum_{t=k}^{\infty} \delta^{t-k} [F(x_t, \bar{\theta}) - F(x_t \wedge x_{t+1}, \bar{\theta})] \\ & - \sum_{t=k}^{\infty} \delta^{t-k} [C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t)] \geq 0. \end{aligned}$$

$F(\cdot, \bar{\theta})$ supermodular: $F(x_t \vee x_{t+1}, \bar{\theta}) - F(x_{t+1}, \bar{\theta})$
 $\geq F(x_t, \bar{\theta}) - F(x_t \wedge x_{t+1}, \bar{\theta}) \quad \forall t \geq k$

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$$\begin{aligned} & C(x_t \vee x_{t+1} - x_{t-1} \vee x_t) - C(x_{t+1} - x_t) \\ & \leq C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t) \quad \forall t \geq k \end{aligned}$$

So (changing variables,) $(x_{t-1} \vee x_t)_{t=1}^{\infty}$ better than $(x_t)_{t=1}^{\infty}$:

$$\begin{aligned} & \sum_{t=k+1}^{\infty} \delta^{t-(k+1)} [F(x_{t-1} \vee x_t, \bar{\theta}) - F(x_t, \bar{\theta})] \\ & - \sum_{t=k+1}^{\infty} \delta^{t-(k+1)} [C(x_{t-1} \vee x_t - x_{t-2} \vee x_{t-1}) - C(x_t - x_{t-1})] \geq 0. \end{aligned}$$

(back to slide 25)

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