# Comparative statics with adjustment costs and the Le Chatelier principle


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# Motivation

Comparative statics: under what circumstances does a parameter shift 'increase' optima / equilibria?

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Comparative statics: under what circumstances does a parameter shift 'increase' optima / equilibria?

Adjustment costs: key feature of many economic models, e.g.

- capital investment (e.g. Jorgenson, 1963; Hayashi, 1982; Cooper & Haltiwanger, 2006)
- sticky prices (e.g. Mankiw, 1985; Caplin & Spulber, 1987; Golosov & Lucas, 2007; Midrigan, 2011)
- trade in illiquid assets (e.g. Kyle, 1985; Back, 1992)
- wealthy hand-to-mouth (e.g. Kaplan & Violante, 2014; Berger & Vavra, 2015; Chetty & Szeidl, 2016)
- labour supply (e.g. Chetty, Friedman, Olsen & Pistaferri, 2011; Chetty, 2012)
- labour demand (e.g. Hamermesh, 1988; Bentolila & Bertola, 1990)

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Adjustment costs: key feature of many economic models.

This paper: comparative statics for adjustment-cost models.

# Example: sticky-price models

Central plank of new Keynesian macro models: sticky prices.

Most important micro-foundation: adjustment ('menu') costs.
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Simplest model: monopolist with constant marg. cost $c \geq 0$,
decr. demand curve $D(\cdot, \eta)$,
parameter $\eta$ shifts |elasticity|.
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decr. demand curve $D(\cdot, \eta)$,
parameter $\eta$ shifts |elasticity|.
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Under what ass'ns on demand $D(\cdot, \eta)$ & adj. cost $C(\cdot)$ do

$$\text{supply shocks } (c \nearrow) \quad \Longrightarrow \quad \text{inflation?}$$
$$\text{demand-elasticity shocks } (\eta \searrow) \quad \Longrightarrow \quad \text{inflation?}$$

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Consequence (Th'm 2): new, greatly generalised Le Chatelier principle.

Consequence (Th'ms 3–6): results extend to infinite-horizon model.

Applications: factor demand, pricing, investment, labour supply, saving.

# Setting

Action $x \in L$, $L \subseteq \mathbf{R}^n$ ($L$ a sublattice)

Objective $F(x, \theta)$ depends on parameter $\theta$

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Parameter $\nearrow$ to $\bar{\theta}$, $\quad$ agent adjusts $x \in L$

Adjusting by $\epsilon = x - \underline{x}$ costs $C(\epsilon) \geq 0$

Agent maximises $G(x, \bar{\theta}) := F(x, \bar{\theta}) - C(x - \underline{x})$.

# Cost assumptions

Cost $\quad C : \Delta L \to [0, \infty] \qquad$ where $\Delta L := \{x - y : x, y \in L\} \subseteq \mathbf{R}^n$.

Assume little about $C$: $\quad C(0) < \infty \quad$ and

- for first result : $C$ <u>minimally monotone</u>
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- for first result : $C$ minimally monotone
- for other results: $C$ monotone

Allows

- non-convex costs (even non-quasiconvex)
- some adjustments infeasible: $C(\epsilon) = \infty$
- non-separability between dimensions (as in Midrigan (2011))

# Minimal monotonicity

$C$ is minimally monotone iff $C(\epsilon \wedge 0) \leq C(\epsilon) \geq C(\epsilon \vee 0) \quad \forall \epsilon.$

('$\wedge$' is entry-wise min; '$\vee$' is entry-wise max.)

Interpretation: cancelling all upward adjustments lowers cost;
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For 1D action $L \subseteq \mathbf{R}$, $C$ minimally monotone
$\iff$ $C$ minimised at 0.

For additively separable $C(\epsilon) = \sum_{i=1}^{n} C_i(\epsilon_i)$,
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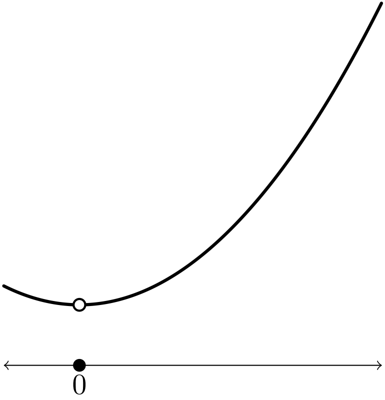
# Minimal monotonicity: example

1D action $L \subseteq \mathbf{R}$

fixed cost $k > 0$

variable cost $a\epsilon^2$ $(a > 0)$

$$C(\epsilon) = \begin{cases} 0 & \text{for} \quad \epsilon = 0 \\ k + a\epsilon^2 & \text{for} \quad \epsilon \neq 0 \end{cases}$$



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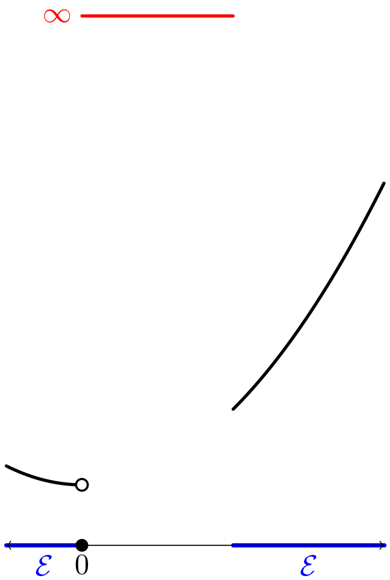

# Review: costless adjustment

**Basic result** (see Milgrom & Shannon, 1994)**:** if $F(x,\theta)$ exhibits

(1) complementarity btw. action $x$ & parameter $\theta$

(2) complementarity btw. action dimensions $x = (x_1, \ldots, x_n)$

then $\bar{x} \geq \underline{x}$ for some $\bar{x} \in \arg\max_{x \in L} F(x, \bar{\theta})$ provided arg max is not empty.

Here

(1) means <u>single-crossing differences in $(x,\theta)$</u>: for any $x' > x$,

$$F(x',\theta) \geq(>) F(x,\theta) \implies F(x',\theta') \geq(>) F(x,\theta') \quad \text{if } \theta < \theta'$$

(2) means <u>quasi-supermodularity in $x$</u>: for any $x, x' \in L$,

$$F(x,\theta) \geq(>) F(x \wedge x',\theta) \implies F(x \vee x',\theta) \geq(>) F(x',\theta) \quad \forall\theta$$

# Review: ordinal vs. cardinal complementarity

SCD & QSM are

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- not inherited by sums: sum of SCD (QSM) functions generally not SCD (QSM)

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- ordinal: preserved by $\nearrow$ transformations
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Cardinal sufficient conditions
that are (not ordinal, but) inherited by sums:

incr. differences in $(x, \theta)$: marginal return $F(x', \theta) - F(x, \theta)$
$\nearrow$ in $\theta$ (for $x' > x$)

supermodularity in $x$: marginal return
$F((x_i', x_j, x_{-ij}), \theta) - F((x_i, x_j, x_{-ij}), \theta)$
$\nearrow$ in $x_j$ (for $x_i' > x_i$, $i \neq j$)

# Comparative statics with costly adjustment

Recall: agent maximises $G(x, \bar{\theta}) \coloneqq F(x, \bar{\theta}) - C(x - \underline{x})$.

**Theorem 1:** if $F(x, \theta)$ exhibits

(1) complementarity btw. action $x$ & parameter $\theta$

(2) complementarity btw. action dimensions $x = (x_1, \ldots, x_n)$

and cost $C$ is minimally monotone,

then $\widehat{x} \geq \underline{x}$ for some $\widehat{x} \in \arg\max_{x \in L} G(x, \bar{\theta})$ $\underset{\text{is not empty}}{\text{provided arg max}}$.

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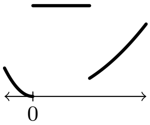
Costs need not even be single-dipped! E.g. $C =$ 


Only <u>ordinal</u> complementarity on $F$. (<u>Not</u> inherited by $G$!)

# Proof of Theorem 1

Fix $x' \in \arg\max_{x \in L} G(x, \bar{\theta})$.

Clearly $\underline{x} \vee x' \geq \underline{x}$.        Will show $\underline{x} \vee x' \in \arg\max_{x \in L} G(x, \bar{\theta})$.

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Standard steps:

$$
\begin{aligned}
& F(\underline{x}, \underline{\theta}) \geq F(\underline{x} \wedge x', \underline{\theta}) && \text{by def'n of } \underline{x} \\
\Longrightarrow \quad & F(\underline{x} \vee x', \underline{\theta}) \geq F(x', \underline{\theta}) && \text{by QSM} \\
\Longrightarrow \quad & F(\underline{x} \vee x', \bar{\theta}) \geq F(x', \bar{\theta}) && \text{by SCD.}
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New step:

$$\begin{aligned} & C\big(\underline{x} \vee x' - \underline{x}\big) \\ = \; & C\big((x' - \underline{x}) \vee 0\big) \\ \leq \; & C\big(x' - \underline{x}\big) && \text{by minimal monotonicity.} \end{aligned}$$

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So

$$\begin{aligned} G(\underline{x} \vee x', \bar{\theta}) &= F(\underline{x} \vee x', \bar{\theta}) - C(\underline{x} \vee x' - \underline{x}) \\ &\geq F(x', \bar{\theta}) - C(x' - \underline{x}) = G(x', \bar{\theta}). \end{aligned}$$

**QED**

# Monotonicity

Cost $C$ is <u>monotone</u> iff for each adj. vector $\epsilon$ & each $i$,

$$C(\epsilon_1, \ldots, \epsilon_{i-1}, \epsilon_i', \epsilon_{i+1}, \ldots, \epsilon_n) \leq C(\epsilon) \quad \text{whenever } 0 \leq \epsilon_i' \leq \epsilon_i$$
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Interpretation: adjusting less lowers cost.

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Interpretation: adjusting less lowers cost.

For $L \subseteq \mathbf{R}$, $C$ monotone

$\iff$ $C$ single-dipped & minimised at 0.

(i.e. $\searrow$ on $(-\infty, 0]$, $\nearrow$ on $[0, \infty)$ )

For additively separable $C(\epsilon) = \sum_{i=1}^{n} C_i(\epsilon_i)$,

$C$ monotone

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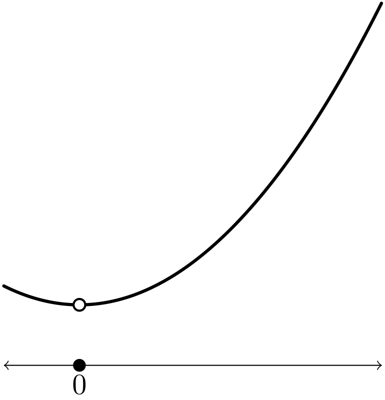
# Example of monotonicity

1D action $L \subseteq \mathbf{R}$

fixed cost $k > 0$

variable cost $a\epsilon^2 \quad (a > 0)$

$$C(\epsilon) = \begin{cases} 0 & \text{for} \quad \epsilon = 0 \\ k + a\epsilon^2 & \text{for} \quad \epsilon \neq 0 \end{cases}$$



# Example of monotonicity violation

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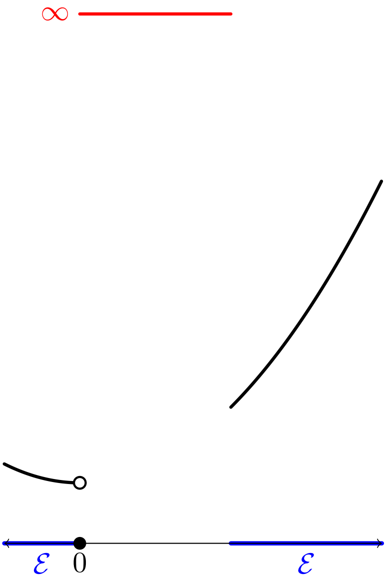

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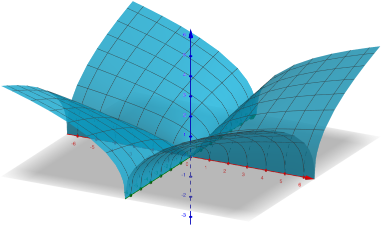
– Euclidean: $C(\epsilon) = \sqrt{\sum_{i=1}^{n} \epsilon_i^2}$.

– Cobb–Douglas: $C(\epsilon) = \prod_{i=1}^{n} |\epsilon_i|^{a_i}$

where $a_1, \ldots, a_n > 0$.

(not quasiconvex)



# The Le Chatelier principle

Le Chatelier principle: $|\text{LR elasticity}| \geq |\text{SR elasticity}|$.

Usual story: only some dimensions of $x$ adjustable in SR.

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Usual story: only some dimensions of $x$ adjustable in SR.

**Formalisation** (Milgrom & Roberts, 1996)**:** suppose $F(x, \theta)$ exhibits

(1) complementarity btw. action $x$ & parameter $\theta$

(2) complementarity btw. action dimensions $x = (x_1, \ldots, x_n)$.

Let $\bar{x} \in \arg\max_{x \in L} F(x, \bar{\theta})$ satisfy $\bar{x} \geq \underline{x}$.[1] (LR optimum.)

Then $\bar{x} \geq \widehat{x} \geq \underline{x}$ for some SR-optimal $\widehat{x}$.

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# General Le Chatelier principle

Different story: SR adjustment is costly.

Nests usual story as (very) special case: $C(\epsilon) = \sum_{i=1}^{n} C_i(\epsilon_i)$,

- some dimensions have $C_i \equiv 0$
- other dimensions have $C_i(\epsilon_i) = \infty$ for every $\epsilon_i \neq 0$.

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**Theorem 2:** suppose $F(x, \theta)$ exhibits

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and cost $C$ is monotone.

Let $\bar{x} \in \arg\max_{x \in L} F(x, \bar{\theta})$ satisfy $\bar{x} \geq \underline{x}$.[2] (LR optimum.)

Then $\bar{x} \geq \widehat{x} \geq \underline{x}$ for some $\widehat{x} \in \arg\max_{x \in L} G(x, \bar{\theta})$ provided arg max is not empty.

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# Proof of Theorem 2

By Theorem 1, may choose $x' \geq \underline{x}$ in $\arg\max_{x \in L} G(x, \bar{\theta})$.

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Standard step: $F(\bar{x} \vee x', \bar{\theta}) \leq F(\bar{x}, \bar{\theta})$ by def'n of $\bar{x}$

$\Longrightarrow$ $F(x', \bar{\theta}) \leq F(\bar{x} \wedge x', \bar{\theta})$ by QSM.

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New step: $x' \geq \bar{x} \wedge x' \geq \underline{x} \quad \Longrightarrow \quad (x' - \underline{x}) \geq (\bar{x} \wedge x' - \underline{x}) \geq 0$

$\Longrightarrow$ $C(x' - \underline{x}) \geq C(\bar{x} \wedge x' - \underline{x})$ by monotonicity.

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So
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\begin{aligned}
G(x', \bar{\theta}) &= F(x', \bar{\theta}) - C(x' - \underline{x}) \\
&\leq F(\bar{x} \wedge x', \bar{\theta}) - C(\bar{x} \wedge x' - \underline{x}) \\
&= G(\bar{x} \wedge x', \bar{\theta}).
\end{aligned}
$$

**QED**

# Why not minimal monotonicity?

Minimal monotonicity is not enough for Le Chatelier.   Example:

$$L = \mathbf{R} \qquad \begin{aligned} F(x,\underline{\theta}) &= -x^2 \\ F(x,\bar{\theta}) &= -(x-2)^2 \end{aligned} \qquad C(\epsilon) = \begin{cases} \infty & \text{if } 0 < \epsilon < 3 \\ 0 & \text{otherwise.} \end{cases}$$


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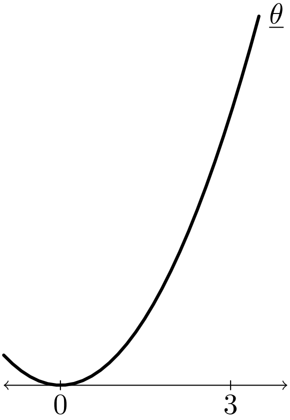

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$$F(x, \bar{\theta}) = -(x-2)^2$$

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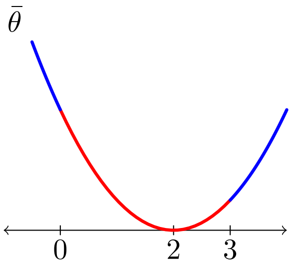
Initial: $\underline{x} = 0 \in \arg\min_{x \in \mathbf{R}} x^2$

SR: $\widehat{x} = 3 \in \arg\min_{x \in \mathbf{R} \setminus (0,3)} (x-2)^2$



# Why not minimal monotonicity?

Minimal monotonicity is not enough for Le Chatelier. Example:

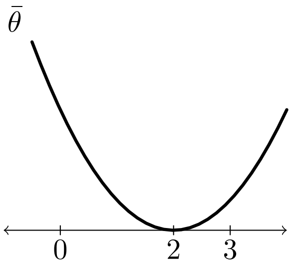
$$L = \mathbf{R} \qquad \begin{aligned} F(x,\underline{\theta}) &= -x^2 \\ F(x,\bar{\theta}) &= -(x-2)^2 \end{aligned} \qquad C(\epsilon) = \begin{cases} \infty & \text{if } 0 < \epsilon < 3 \\ 0 & \text{otherwise.} \end{cases}$$

$C$ is minimally monotone,
not monotone.

Initial: $\underline{x} = 0 \in \arg\min_{x\in\mathbf{R}} x^2$

SR: $\widehat{x} = 3 \in \arg\min_{x\in\mathbf{R}\setminus(0,3)} (x-2)^2$

LR: $\bar{x} = 2 \in \arg\min_{x\in\mathbf{R}} (x-2)^2$



# Application to factor demand

Inputs $(k, \ell)$, real input prices $(r, w)$, production function $f$

Monotone adjustment cost

Profit $F(k, \ell, -w) = f(k, \ell) - rk - w\ell$

- supermodular in $(k, \ell)$: if $f$ supermodular (complements)
- incr. diff. in $((k, \ell), -w)$: $\nabla_{(k,\ell)} F = \binom{f_k - r}{f_\ell - w}$ $\nearrow$ in $-w$

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As $w \searrow$, both factor demands $\nearrow$ (Theorem 1)

In LR, both factor demands further $\nearrow$ (Theorem 2)

If instead $f$ submodular (substitutes), $\ell$ demand $\nearrow$ but
$k$ demand $\searrow$.

# Application to pricing

Menu-cost model: monopolist with constant marg. cost $c \geq 0$,

decr. demand curve $D(\cdot, \eta)$,

parameter $\eta$ shifts |elasticity|.

Adjusting price by $\epsilon$ costs $C(\epsilon) \geq 0$.

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Adjusting price by $\epsilon$ costs $C(\epsilon) \geq 0$.

Profit $F(p,(c,-\eta)) = (p-c)D(p,\eta)$

– $F$ QSM in $p$: automatic since $p$ one-dimensional ($\in \mathbf{R}$)

– $\ln F$ has incr. differences in $(p,(c,-\eta))$:

$$\frac{\mathrm{d}}{\mathrm{d}p}\ln F = \frac{1}{p-c} + \frac{D'(p,\eta)}{D(p,\eta)} = \frac{1}{p-c} - \frac{|\text{elasticity}(p,\eta)|}{p}$$

$\nearrow$ in $c$ & $-\eta$

# Dynamic adjustment

Agent chooses $x_t \in L$ in each period $t \in \mathbf{N}$

Period-$t$ payoff: $F(x_t, \theta_t) - C_t(x_t - x_{t-1})$

Taken as given:

- initial choice $x_0 = \underline{x} \in \arg\max_{x \in L} F(x, \underline{\theta})$
- parameter path $(\theta_t)_{t=1}^{\infty}$
- cost path $(C_t)_{t=1}^{\infty}$

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Agent forward-looking, discount rate $\delta \in (0, 1)$

Chooses path $(x_t)_{t=1}^{\infty}$ to max $\displaystyle\sum_{t=1}^{\infty} \delta^t [F(x_t, \theta_t) - C_t(x_t - x_{t-1})].$

# Dynamic Le Chatelier principle

**Theorem 3:** suppose $F(x, \theta)$ exhibits

(1) complementarity btw. action $x$ & parameter $\theta$

(2) complementarity btw. action dimensions $x = (x_1, \ldots, x_n)$,

and that each cost $C_t$ is monotone.

Fix $\bar{x} \in \arg\max_{x \in L} F(x, \bar{\theta})$ such that $\bar{x} \geq \underline{x}$.[3]

If $\underline{\theta} \leq \theta_t \leq \bar{\theta} \quad \forall t,$ then $\underline{x} \leq x_t \leq \bar{x} \quad \forall t$

for some solution $(x_t)_{t=1}^{\infty}$, provided a solution exists.

Proof: straightforward extension of Theorem 1+2 logic.

[3]Such $\bar{x}$ exists by the 'basic result' (sl. 9), provided argmax nonempty.

# Strong dynamic Le Chatelier principle

**Theorem 4:** suppose $F(x,\theta)$ exhibits

(1) complementarity btw. action $x$ & parameter $\theta$

(2) cardinal complementarity btw. dimensions $x=(x_1,\ldots,x_n)$

(3) boundedness in $x$ on each compact set $\subseteq L$, for each $\theta$,

and that $C_t = C \;\; \forall t$ for $C$ monotone & additively separable.

Fix $\bar{x} \in \arg\max_{x \in L} F(x,\bar{\theta})$ such that $\bar{x} \geq \underline{x}$.[4]

If $\theta_t = \bar{\theta} \quad \forall t,$ then $\underline{x} \leq x_t \leq x_{t+1} \leq \bar{x} \quad \forall t$

for some solution $(x_t)_{t=1}^{\infty}$, provided a solution exists.

[4] Such $\bar{x}$ exists by the 'basic result' (sl. 9), provided argmax nonempty.

# Sketch proof of Theorem 4

Can 'monotonise' any $(x_t)_{t=1}^{\infty}$ by replacing $t^{\text{th}}$ entry with cumulative max $x_1 \vee x_2 \vee \cdots \vee x_{t-1} \vee x_t$.

Claim: monotonisation preserves optimality.

(Suffices since can then monotonise solution from Theorem 3.)

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Claim: monotonisation preserves optimality.

(Suffices since can then monotonise solution from Theorem 3.)

By boundedness & limit argument, suffices to show that if $(x_t)_{t=1}^{\infty}$ optimal & $x_1 \leq x_2 \leq \cdots \leq x_{k-1} \leq x_k$, remains optimal if replace $t^{\text{th}}$ entry by $x_{t-1} \vee x_t$ $\forall t \geq k+1$.

Proved using supermodularity of $F(\cdot, \bar{\theta})$,
+ monotonicity & additive separability of $C$.

(argument: slide 28)

# Application to pricing, continued

Assume cost $C$ time-invariant,
demand $D(\cdot, \eta)$ upper semi-continuous.

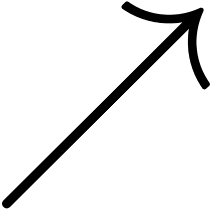
Profit $F(p,(c,-\eta)) = (p-c)D(p,\eta)$

- $F(p,(c,-\eta))$ SM in $p$ & $C$ additively separable: automatic since $p$ one-dimensional ($\in \mathbf{R}$)
- $F(p,(c,-\eta))$ bounded in $p$ on each compact set $\subseteq \mathbf{R}_+$

Theorem 4:

- supply shock ($c \nearrow$) $\Longrightarrow$ inflation at every horizon
- demand shock s.t. $\eta \searrow$ $\Longrightarrow$ inflation at every horizon

# Thanks!



# Sketch proof of Theorem 4: main step

$(x_t)_{t=1}^{\infty}$ optimal $\implies$ better than $(x_t \wedge x_{t+1})_{t=1}^{\infty}$:

$$\sum_{t=k}^{\infty} \delta^{t-k} \left[ F(x_t, \bar{\theta}) - F(x_t \wedge x_{t+1}, \bar{\theta}) \right]$$
$$- \sum_{t=k}^{\infty} \delta^{t-k} [C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t)] \geq 0.$$

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$$-\sum_{t=k}^\infty \delta^{t-k}[C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t)] \geq 0.$$

$F(\cdot,\bar{\theta})$ supermodular:

$$F(x_t \vee x_{t+1},\bar{\theta}) - F(x_{t+1},\bar{\theta})$$
$$\geq F(x_t,\bar{\theta}) - F(x_t \wedge x_{t+1},\bar{\theta}) \qquad \forall t \geq k$$

$C$ monotone & additively separable: (argument omitted)

$$C(x_t \vee x_{t+1} - x_{t-1} \vee x_t) - C(x_{t+1} - x_t)$$
$$\leq C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t) \qquad \forall t \geq k$$

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$(x_t)_{t=1}^{\infty}$ optimal $\implies$ better than $(x_t \wedge x_{t+1})_{t=1}^{\infty}$:

$$\sum_{t=k}^{\infty} \delta^{t-k}\left[F(x_t,\bar{\theta}) - F(x_t \wedge x_{t+1},\bar{\theta})\right]$$
$$- \sum_{t=k}^{\infty} \delta^{t-k}\left[C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t)\right] \geq 0.$$

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$$F(x_t \vee x_{t+1},\bar{\theta}) - F(x_{t+1},\bar{\theta})$$
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$C$ monotone & additively separable: (argument omitted)

$$C(x_t \vee x_{t+1} - x_{t-1} \vee x_t) - C(x_{t+1} - x_t)$$
$$\leq C(x_t - x_{t-1}) - C(x_t \wedge x_{t+1} - x_{t-1} \wedge x_t) \qquad \forall t \geq k$$

So (changing variables,) $(x_{t-1} \vee x_t)_{t=1}^{\infty}$ better than $(x_t)_{t=1}^{\infty}$:

$$\sum_{t=k+1}^{\infty} \delta^{t-(k+1)}\left[F(x_{t-1} \vee x_t,\bar{\theta}) - F(x_t,\bar{\theta})\right]$$
$$- \sum_{t=k+1}^{\infty} \delta^{t-(k+1)}\left[C(x_{t-1} \vee x_t - x_{t-2} \vee x_{t-1}) - C(x_t - x_{t-1})\right] \geq 0.$$

(back to slide 25)

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