The comparative statics of persuasion

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Motivation

Canonical persuasion model (Kamenica & Gentzkow, 2011)

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model: a sender designs signal.

(no functional-form restrictions)

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- more applic'ns: grades

  labelling (food labels, energy ratings, ...)

  credit scores

  ...

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Canonical persuasion model (Kamenica & Gentzkow, 2011)

Main question: 'what are optimal signals like?' Hard.

e.g. Kolotilin (2014, 2018), Gentzkow and Kamenica (2016), Dworczak and Martini (2019), Kleiner, Moldovanu and Strack (2021), Arieli, Babichenko, Smorodinsky and Yamashita (2023)

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Open question: 'how do optimal signals vary with primitives?'

This paper: answer that question.

Overview

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$\hookrightarrow$ special^2 cases: known comparative-statics results

(Kolotilin, Mylovanov and Zapechelnyuk, 2022; Gitmez and Molavi, 2023)

Plan

The persuasion model

‘Non-decreasing’ comparative statics

‘Increasing’ comparative statics

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Terminology: 'distribution' means CDF $[0,1] \to [0,1]$.

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Assumption: sender cares only about posterior mean.
Payoff $u(m)$ from posterior mean $m \in [0,1]$.

$\hookrightarrow$ motivated by applications; common in recent lit.

Sender chooses signal to max $\mathbf{E}\big[u(\text{random posterior mean})\big]$.

Interpretation

$u(\cdot)$ is a reduced-form object.

Captures (expected) payoff from downstream interaction.

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Captures (expected) payoff from downstream interaction.

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Our analysis is robust to downstream details:
identifies necessary & sufficient conditions directly on u.

$\hookrightarrow$ can then check these in applications.

Application: privately informed receiver

Model of Kolotilin, Mylovanov, Zapechelnyuk and Li (2017):

Receiver chooses whether to ‘participate’; sender hopes yes.

$\hookrightarrow$ example: whether to buy sender’s good.

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Outside option worth $R \sim G$, privately observed by receiver.

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

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Question: what shifts of G cause more info-provision?

Kolotilin's (2014) reformulation

Model: $\max_{S \in \{\text{signals}\}} \mathbf{E}_S[u(\text{random posterior mean})]$

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Reformulation: sender chooses F_S directly.

Optimal choices: $\arg\max_{F \text{ feasible given } F_0} \int u \mathrm{d} F$

where 'F feasible given F_0'

$\overset{\text{def'n}}{\iff}$ $\exists$ signal S such that $F_S = F$.

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Fact: F feasible given F_0
$\iff$ F a mean-preserving contraction of F_0
$\left(\overset{\text{def'n}}{\iff} \int_0^x F \leq \int_0^x F_0 \quad \forall x \in [0, 1) \quad \& \quad \int_0^1 F = \int_0^1 F_0 \right)$.

Informativeness

Definition: F is <u>less informative</u> than G
iff $\int \psi \mathrm{d}F \leq \int \psi \mathrm{d}G$ for every convex $\psi : [0,1] \to \mathbf{R}$.

In the spirit of D. Blackwell.

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Fact: F less informative than G
$\iff$ F a mean-preserving contraction of G.

'Less informative' is demanding:

frequently F is not less informative than G <u>and</u>
G is not less informative than F.

More comparisons

$$F \ \underline{\text{strictly less informative}} \text{ than } G$$
$$\overset{\text{def'n}}{\iff} \quad F \text{ less informative than } G \quad \& \quad F \neq G.$$

$$G \ \underline{\text{(str.) more informative}} \text{ than } F$$
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In principle, argmax can have ≥ 2 elements

$\implies$ must compare <u>sets</u> of dist'ns.

This talk: assume all argmaxes singleton.

'Increasing' comparative statics

Question : for interim payoffs $u, v : [0,1] \to \mathbf{R}$,
what must we assume to conclude that

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \qquad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \qquad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F$$

whatever the prior F_0?

‘Non-decreasing’ comparative statics

‘Increasing’ is a lot to ask. Begin with non-decreasing:

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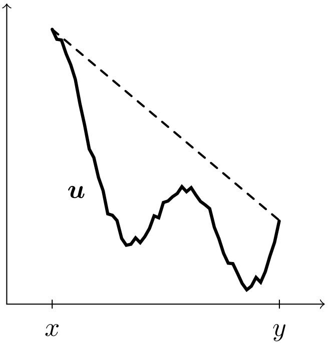
for any $x < y$ in $[0,1]$ such that

$u(\alpha x + (1-\alpha) y) \leq \alpha u(x) + (1-\alpha) u(y)$

holds $\forall\, \alpha \in (0,1)$



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holds $\forall\, \alpha \in (0,1)$,

$v(\alpha x + (1-\alpha)y) \leq \alpha v(x) + (1-\alpha)v(y)$

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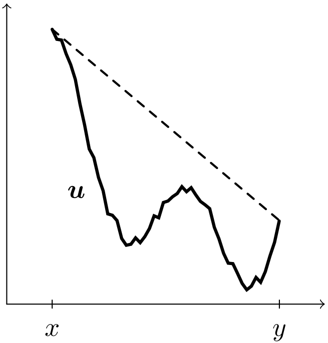

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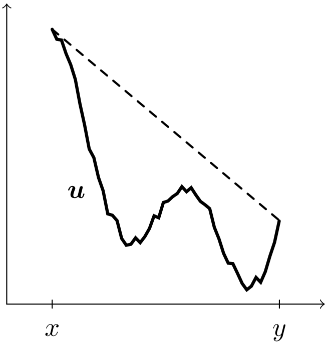
and for each α, former ineq. strict $\Longrightarrow$ latter ineq. strict.



Sufficient conditions

Lemma: if $v(x) = \Phi(u(x), x) \quad \forall x$
where Φ convex & $\Phi(\cdot, x)$ str. incr. $\forall x$,
then u is coarsely less convex than v.

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Lemma: if $v(x) = \Phi(u(x), x) \quad \forall x$
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Proof: $u(\alpha x + (1-\alpha)y) \leq(<) \alpha u(x) + (1-\alpha)u(y) \qquad \Longrightarrow$

$$v(\alpha x + (1-\alpha)y) \leq(<) \Phi\Big(\alpha u(x) + (1-\alpha)u(y), \alpha x + (1-\alpha)y\Big)$$
$$\leq \alpha v(x) + (1-\alpha)v(y)$$

by str. monotonicity & convexity. ■

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Special case:
(usual 'less convex than')

$v = \phi \circ u$ for a convex
& str. incr.
$\phi : \mathbf{R} \to \mathbf{R}$

$$\left(\iff \quad \begin{array}{l} u'' \cdot |v'| \leq v'' \cdot |u'| \\ \text{if } u, v \text{ are } C^2 \end{array} \right)$$

$\hookrightarrow$ take $\Phi(k, x) = \phi(k)$.

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Special case:
(from costly info acq. lit)

$v = u + \psi$ for a convex $\psi : [0,1] \to \mathbf{R}$

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$\hookrightarrow$ take $\Phi(k, x) = k + \psi(x)$.

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

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G improves in MLR sense e.g. $\mu \nearrow$ if $G = N(\mu, \sigma^2)$

$$\overset{\text{def'n}}{\Longleftrightarrow} \quad g'/g \quad \nearrow \text{ pointwise}$$

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So by Lemma, improved outside-option dist'n G
$\Longrightarrow$ coarsely more convex u.

'Non-decreasing' comparative statics

Theorem 1: For upper semi-continuous $u, v : [0,1] \to \mathbf{R}$, the following are equivalent:

- u is coarsely less convex than v.
- For any prior dist'n F_0,

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \substack{\text{is not str. more} \\ \text{info'tive than}} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F.$$

Proof idea

Th'm 1: For usc u & v, u is coarsely less convex than v iff

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<u>Necessity</u> of 'u coarsely less convex than v': straightforward.

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Necessity of 'u coarsely less convex than v': straightforward.

Sufficiency: u coarsely less convex than v

$\implies$ $U(F) \coloneqq \int u \mathrm{d}F$ interval-dominated by $V(F) \coloneqq \int v \mathrm{d}F$

1st implication: non-trivial.

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Necessity of 'u coarsely less convex than v': straightforward.

Sufficiency: u coarsely less convex than v

$\Longrightarrow$ $U(F) := \int u \mathrm{d}F$ interval-dominated by $V(F) := \int v \mathrm{d}F$

$$\Longrightarrow \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} U(F) \quad \begin{array}{c}\text{is not str. more}\\ \text{info'tive than}\end{array} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} V(F)$$

1st implication: non-trivial.

2nd implication: a theorem of Quah and Strulovici (2009, 2007).

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'Increasing' comparative statics

Halfway there

By Theorem 1, 'more convexity' is necessary & <u>not</u> sufficient for <u>increasing</u> comparative statics.

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What can go wrong? Example at end of talk (if time allows).

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Remaining question: what further restriction on u is needed?

Regularity

From now on, focus on regular u.

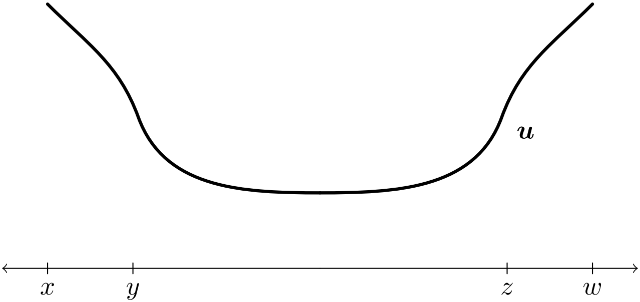
'Regular': slightly weaker than twice contin'sly differentiable.

(def'n: slide 33)

Crater property

Definition: regular $u : [0,1] \to \mathbf{R}$ sat's the crater property iff

$$\forall \;\; x < y < z < w \quad \text{s.t.} \quad u \begin{cases} \text{concave} & \text{on } [x,y] \\ \text{str. convex} & \text{on } [y,z] \\ \text{concave} & \text{on } [z,w], \end{cases}$$



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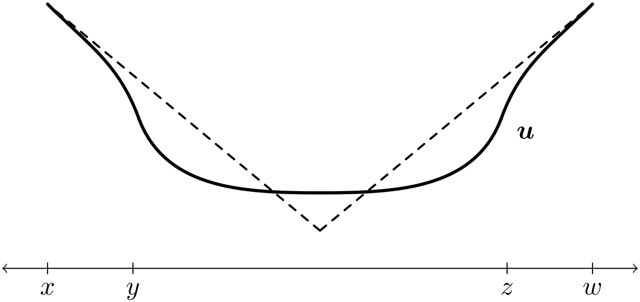
have $u'(x) \neq u'(w)$



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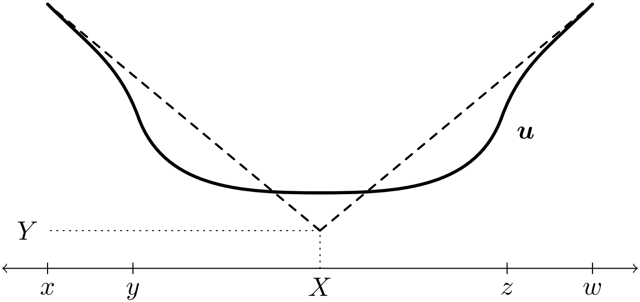
have $u'(x) \neq u'(w)$, & tangents at x & at w cross at (X, Y)

s.t. (i) $y \leq X \leq z$ & (ii) $Y \leq u(X)$.



(satisfied)

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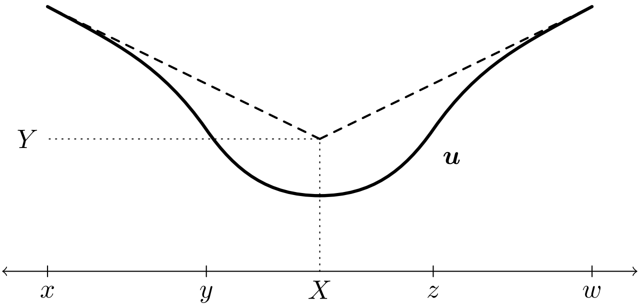
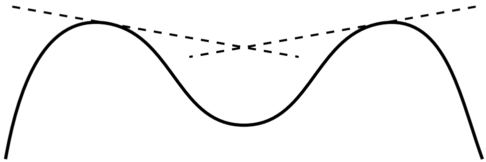


(violated)

When does the crater property hold?

Crater property is strong.

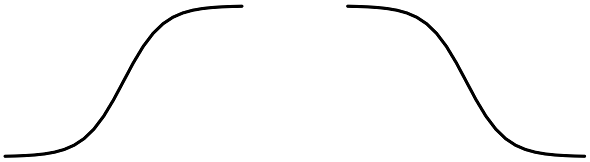
↪ e.g. rules out multiple interior local maxima.



When does the crater property hold?

Sufficient conditions:

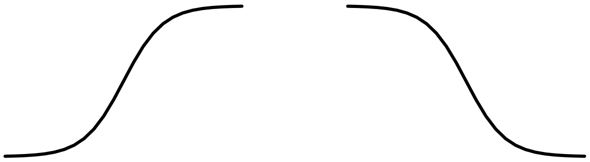
– ‘S’ shape: str. convex–concave or concave–str. convex.



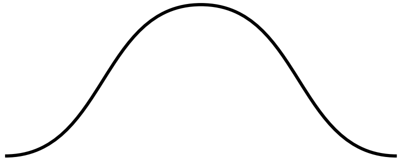
When does the crater property hold?

Sufficient conditions:

– ‘S’ shape: str. convex–concave or concave–str. convex.



– ‘bell’ shape: str. convex–concave–str. convex.



Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

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G unimodal e.g. $G = N(\mu, \sigma^2)$

$$\overset{\text{def'n}}{\iff} \quad g \begin{Bmatrix} \text{str. incr.} & \text{on } [0, x] \\ \text{str. decr.} & \text{on } [x, 1] \end{Bmatrix} \quad \text{for some } x$$

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$$\iff \quad G \begin{Bmatrix} \text{str. convex} & \text{on } [0, x] \\ \text{str. concave} & \text{on } [x, 1] \end{Bmatrix} \quad \text{for some } x$$

$\implies$ u S-shaped $\implies$ u obeys crater property.

‘Increasing’ comparative statics

Theorem 2: For a regular $u : [0,1] \to \mathbf{R}$,
the following are equivalent:

– u satisfies the crater property.

– For every regular & coarsely more convex $v : [0,1] \to \mathbf{R}$ and every atomless convex-support F_0,

$$\mathop{\arg\max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \substack{\text{is less} \\ \text{info'tive than}} \quad \mathop{\arg\max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F.$$

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Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\Longrightarrow \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\Longrightarrow$ u S-shaped
$\Longrightarrow$ u obeys crater property.

Application: privately informed receiver

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Recall: G improves in MLR sense
$\Longleftrightarrow$ G becomes more convex (in usual sense)
$\Longrightarrow$ u becomes coarsely more convex.

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Recall: G improves in MLR sense
$\iff$ G becomes more convex (in usual sense)
$\implies$ u becomes coarsely more convex.

By Th'm 2, G unimodal & improves in MLR sense
$\implies$ sender provides more info ($\forall$ prior).

$\hookrightarrow$ recovers Prop 1 in Kolotilin, Mylovanov and Zapechelnyuk (2022)

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\implies$ u S-shaped
$\implies$ u obeys crater property.

More generally, if G improves. E.g. $g' \nearrow$ pointwise
$\iff$ $G'' \nearrow$ pointwise
$\implies$ u coarsely more c'vex.

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\implies$ u S-shaped
$\implies$ u obeys crater property.

Alternatively: if G becomes 'more diffuse' in sense that
g becomes less convex (in usual sense).

↪ generalises Gitmez and Molavi (2023),
who assume binary prior

Proof of sufficiency

Th'm 2: A regular u obeys crater property iff

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{matrix}\text{is less}\\ \text{info'tive than}\end{matrix} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F$$

$\forall$ regular coarsely more c'vex v, $\forall$ atomless c'vex-supp't F_0.

Bespoke argument, relies on persuasion structure.

$\hookrightarrow$ study the dual (Dworczak & Martini, 2019)

Proof of sufficiency

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$\forall$ regular coarsely more c'vex v, $\forall$ atomless c'vex-supp't F_0.

Bespoke argument, relies on persuasion structure.

$\hookrightarrow$ study the dual (Dworczak & Martini, 2019)

Cannot use general comparative-statics results:

they require $U(F) = \int u \mathrm{d}F$ (interval-)quasi-supermodular

which is super-strong (requires u concave or u str. convex)

(sketch proof of necessity: slide 34)

Robustness & extensions

– restricted classes of priors F_0 (slide 35)

– 'decreasing' comparative statics (slide 37)

– constrained persuasion (slide 38)

– shifts of the prior F_0 (slide 39)

Application: alignment of interests

Question: alignment $\nearrow$ $\Longrightarrow$ info-provision $\nearrow$?

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Setting: actions $a \in \mathcal{A}$, payoffs $U_S(a, m)$, $U_R(a, m)$,
choice $A(m)$ U_R-optimal $\Big(\in \arg\max_{a \in \mathcal{A}} U_R(a, m)\Big)$
$\implies u(m) = U_S\big(A(m), m\big)$.

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Example: shift from $(a,m) \mapsto U_S(a,m)$
to $(a,m) \mapsto U_S(a,m) + \phi\Big(U_R(a,m)\Big)$
where ϕ str. incr. (' alignment $\nearrow$ ')

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choice $A(m)$ U_R-optimal $\Big(\in \arg\max_{a \in \mathcal{A}} U_R(a,m)\Big)$
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to $(a,m) \mapsto U_S(a,m) + \phi\Big(U_R(a,m)\Big)$
where ϕ str. incr. (' alignment $\nearrow$ ')

ϕ convex: u becomes coarsely more convex.
$\forall$ U_S, U_R, & U_R-optimal $A(\cdot)$

(general: slide 40)

ϕ concave: u may become coarsely less convex!
$\exists$ U_S, U_R, & U_R-optimal $A(\cdot)$

Conclusion

Open question in canonical persuasion model:

when does a shift of model parameters cause sender to choose a more informative signal?

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Complete answer:

u obeys crater property + becomes coarsely more convex.

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Open question in canonical persuasion model:

when does a shift of model parameters cause
sender to choose a more informative signal?

Complete answer:

u obeys crater property + becomes coarsely more convex.

Applied upshot:

– easy-to-check sufficient conditions

– applications (see paper)

Conclusion

Remaining questions:

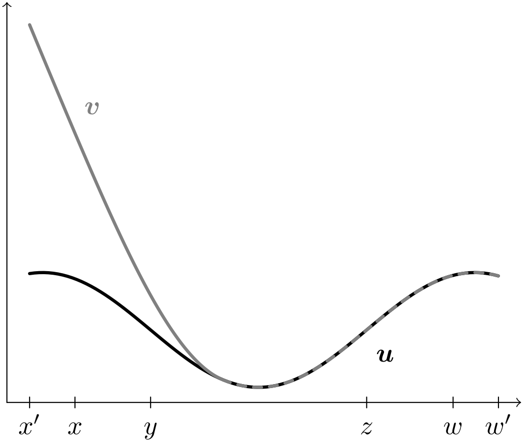
- further applications

Conclusion

Remaining questions:

- further applications
- case when ≥ 2 moments matter (not just mean).

Thanks!



Application: details

Detail 1: assume $R \perp\!\!\!\perp$ (value of particip'n).

Detail 2: Can sender do better by offering a menu of signals?
No. (Kolotilin, Mylovanov, Zapechelnyuk & Li, 2017, Th'm 1)

(back to slide 7)

Regularity: definition

Definition: $u : [0,1] \to \mathbf{R}$ is <u>regular</u> iff both

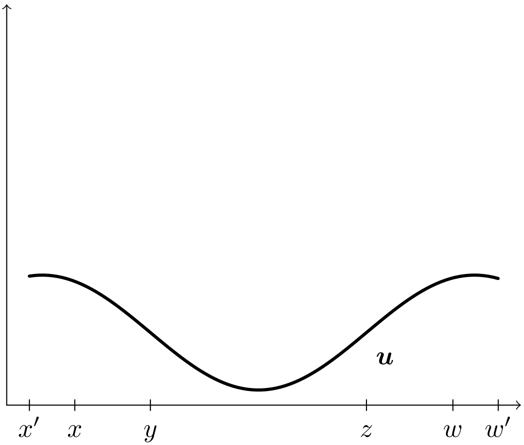
(i) u is contin's & possesses contin's & bounded derivative $u' : (0,1) \to \mathbf{R}$

(ii) $[0,1]$ may be partitioned into finitely many intervals on which u is either affine, str. convex, or str. concave.

Sufficient condition: u twice contin'sly differentiable.

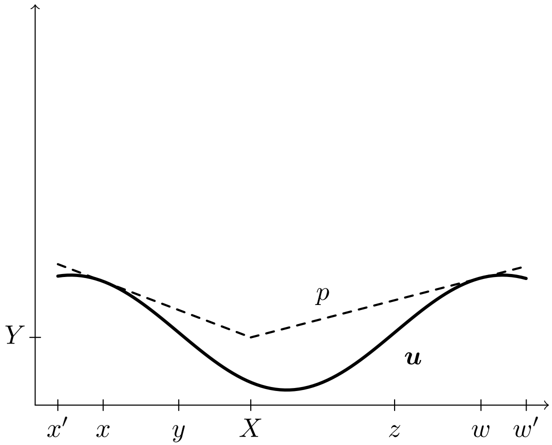
(back to slide 21)

Sketch proof of necessity



Suppose u regular & violates crater.

Sketch proof of necessity



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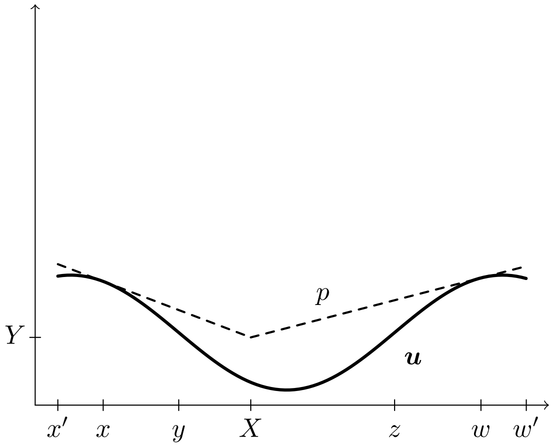

Suppose u regular & violates crater.

Construct F_0:

- atomless
- support $[x', w']$
- $\frac{\int_0^X \xi F_0(\mathrm{d}\xi)}{F_0(X)} = x$
- $\frac{\int_X^1 \xi F_0(\mathrm{d}\xi)}{1-F_0(X)} = w.$

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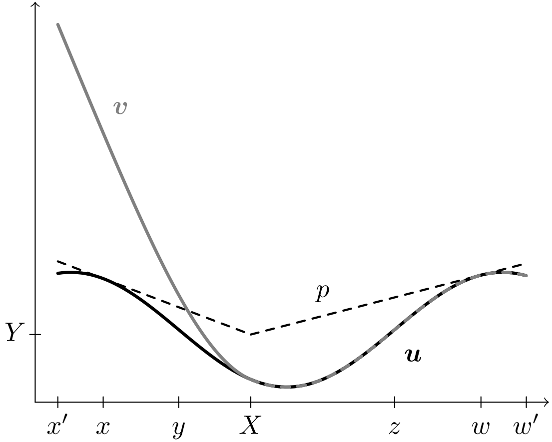

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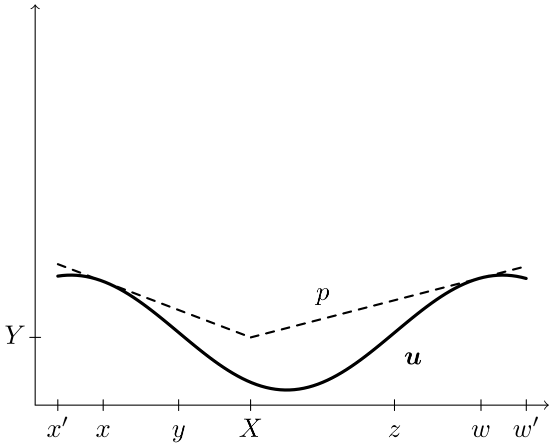
Construct v:

– on $[0, X]$, $\geq u$ & str. convex

– on $[X, 1]$, $= u.$

Sketch proof of necessity



For u, optimal dist'n F reveals (only) whether state $\gtrless X$.

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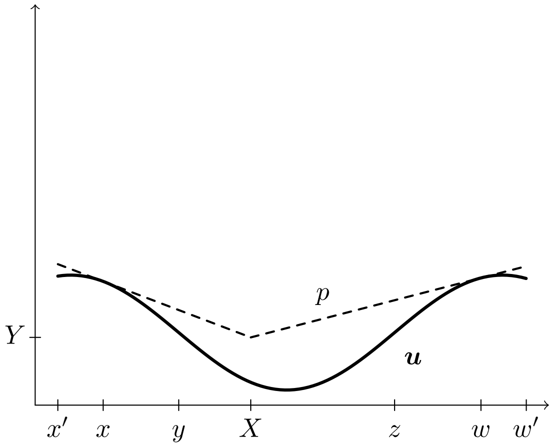

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no pooling acr. X.

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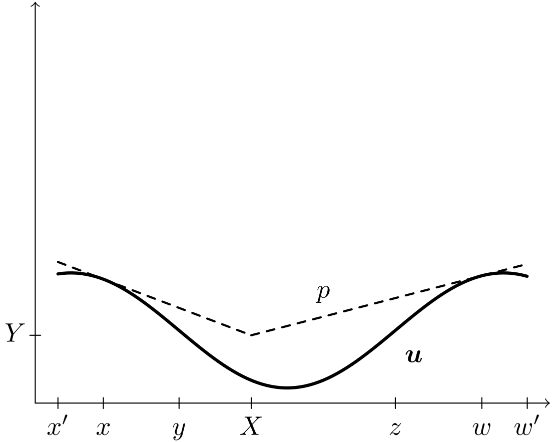


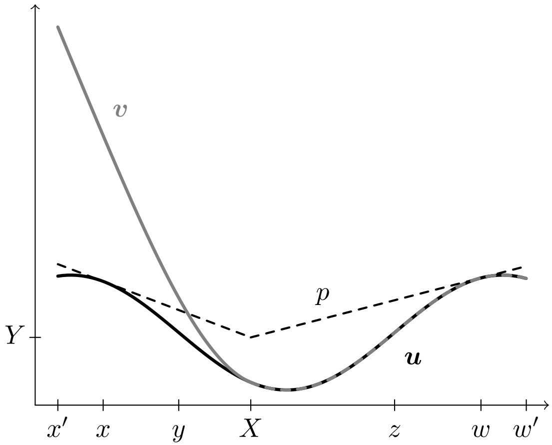
For u, optimal dist'n F reveals (only) whether state $\gtrless X$:

no pooling acr. X.

Proof: $\forall\, H$ less info. than F_0,

$$\begin{aligned} & \int u \mathrm{d}F \\ = & \int p \mathrm{d}F && u \overset{F\text{-a.e.}}{=} p \\ = & \int p \mathrm{d}F_0 && p \text{ aff. } [0,X] \ \&\ [X,1] \\ \geq & \int p \mathrm{d}H && p \text{ convex} \\ \geq & \int u \mathrm{d}H && p \geq u. \quad \blacksquare \end{aligned}$$

Sketch proof of necessity



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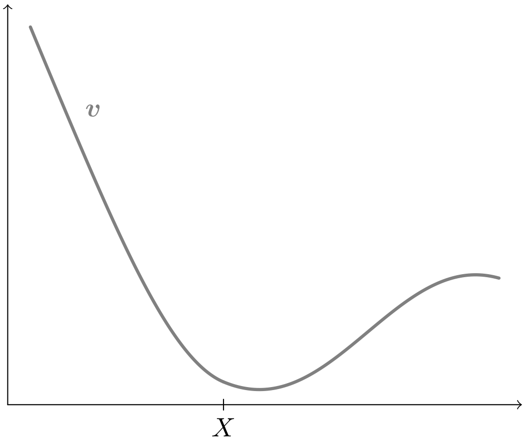
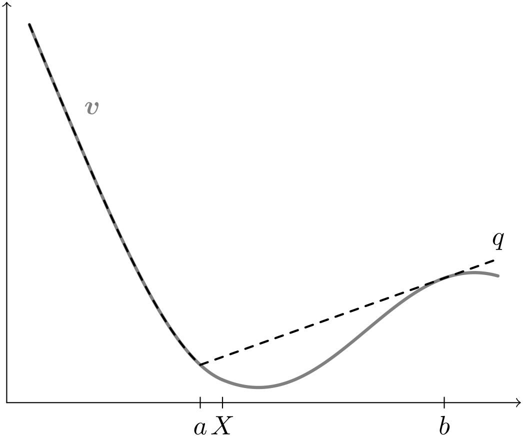


v S-shaped.

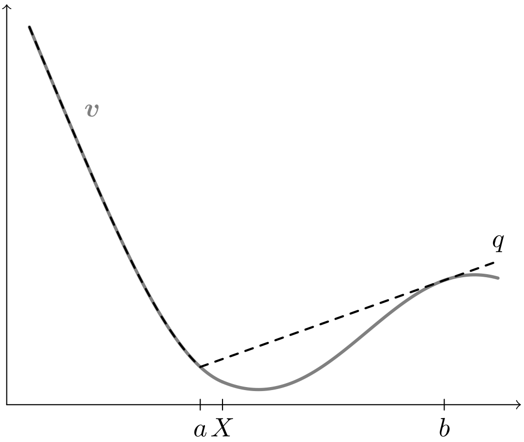
Sketch proof of necessity



v S-shaped $\implies$ optimal dist'n G reveals $[0, a)$, pools $[a, 1]$.

$$\text{where} \quad b = \frac{1}{1 - F_0(a)} \int_a^1 \xi F_0(\mathrm{d}\xi)$$

Sketch proof of necessity



v S-shaped $\implies$ optimal dist'n G reveals $[0, a)$, pools $[a, 1]$:

so pools across X.

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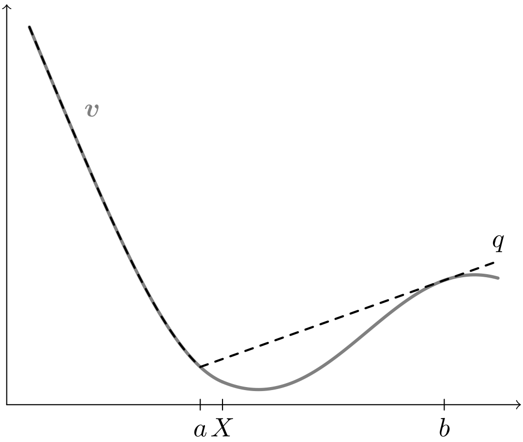


where $\quad b = \dfrac{1}{1 - F_0(a)} \displaystyle\int_a^1 \xi F_0(\mathrm{d}\xi)$

v S-shaped $\implies$ optimal dist'n G reveals $[0, a)$, pools $[a, 1]$:

so pools across X.

Proof: $\forall\, H$ less info. than F_0,

$$\begin{aligned} & \textstyle\int v \mathrm{d}G \\ = & \textstyle\int q \mathrm{d}G && v \overset{G\text{-a.e.}}{=} q \\ = & \textstyle\int q \mathrm{d}F_0 && q \text{ aff. } [a,1] \\ \geq & \textstyle\int q \mathrm{d}H && q \text{ convex} \\ \geq & \textstyle\int v \mathrm{d}H && q \geq v. \quad \blacksquare \end{aligned}$$

(back to slide 27)

Restricted classes of priors

Th'm 2: crater property necessary if consider <u>all</u> priors F_0.

Restricted classes of priors

Th'm 2: crater property necessary if consider <u>all</u> priors F_0.

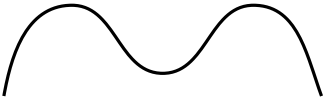
Robustness: necessary even if consider only a <u>single</u> F_0:

Prop'n: Provided $|\operatorname{supp} F_0| \geq 3$, $\exists$ regular $u, v : [0,1] \to \mathbf{R}$ such that u is coarsely less convex than v, but

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{matrix}\text{is not less}\\ \text{info'tive than}\end{matrix} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F.$$

Can choose u M-shaped & v S-shaped.

'M-shaped' = concave–str. convex–concave.



($|\operatorname{supp} F_0| \leq 2$: next slide) (back to slide 28)

Binary priors

Binary prior: F_0 with $|\operatorname{supp} F_0| \leq 2$.

Effectively: state is binary.

Binary priors

Binary prior: F_0 with $|\operatorname{supp} F_0| \leq 2$.

Effectively: state is binary.

Binary priors are special—no need for crater property:

Prop'n: For upper semi-continuous $u, v : [0,1] \to \mathbf{R}$, the following are equivalent:

- u is coarsely less convex than v.
- For any binary prior dist'n F_0,

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{matrix}\text{is less}\\ \text{info'tive than}\end{matrix} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F.$$

(back to slide 28)

‘Decreasing’ comparative statics

Symmetric counterpart to question answered by Th’m 2:

what ass’ns on v ensure comparative statics

with any coarsely less convex u, whatever the prior F_0?

'Decreasing' comparative statics

Symmetric counterpart to question answered by Th'm 2:

what ass'ns on v ensure comparative statics

with any coarsely less convex u, whatever the prior F_0?

Answer: need super-strong ass'ns:

Prop'n: For a regular $v : [0,1] \to \mathbf{R}$,
the following are equivalent:

- v is either concave or str. convex.
- For every regular & coarsely less convex $u : [0,1] \to \mathbf{R}$ and every atomless convex-support F_0,

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F.$$

(back to slide 28)

Constrained persuasion

Sender may face constraints on choice of signal. Growing lit.

Two natural constraints:

– only monotone partitional signals

– only signals that send $\leq K$ messages, for some $K \in \mathbf{N}$

Prop'n: in both cases, crater property remains necessary.

(back to slide 28)

Shifts of the prior

Shifts of prior F_0 instead of payoff u.

Interpret'n: change in info available to sender.

Shifts of the prior

Shifts of prior F_0 instead of payoff u.

Interpret'n: change in info available to sender.

Prop'n: there are no $F_0 \neq G_0$ such that

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{matrix}\text{is less} \\ \text{info'tive than}\end{matrix} \quad \operatorname*{arg\,max}_{F \text{ feas. given } G_0} \int u \mathrm{d}F$$

for every regular and S-shaped $u : [0,1] \to \mathbf{R}$.

Upshot: comparative statics highly u-sensitive.
No result across all u, not even all S-shaped u.

(back to slide 28)

Application: alignment of interests, in general

Alignment $\nearrow$: shift from $(a, m) \mapsto U_S(a, m)$

to $(a, m) \mapsto \Phi\Big(U_S(a, m), U_R(a, m), m\Big)$

where Φ an alignment-incr'ing utility transform'n (AIUT):

- <u>utility transformation:</u> $\Phi(\cdot, \ell, m)$ str. incr. $\forall\ \ell, m$
- <u>alignment-increasing:</u> $\Phi(k, \cdot, m)$ incr. $\forall\ k, m.$

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– alignment-increasing: $\Phi(k, \cdot, m)$ incr. $\forall\, k, m$.

Prop'n: For any convex AIUT Φ,

$m \mapsto U_S(A(m), m)$ is coarsely less convex than

$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$

$\forall\ U_S,\ U_R,$ $\forall\ U_R$-optimal $A(\cdot)$.

Application: alignment of interests, in general

AIUT: Φ such that $\Phi(\cdot, \ell, m)$ str. incr. & $\Phi(k, \cdot, m)$ incr.

Prop'n: $\forall$ convex AIUT Φ, $\forall\ U_S, U_R$, $\forall\ U_R$-optimal $A(\cdot)$,
$m \mapsto U_S(A(m), m)$ is coarsely less convex than
$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$.

Convexity is essential. (Nearly necessary.)

Application: alignment of interests, in general

AIUT: Φ such that $\Phi(\cdot, \ell, m)$ str. incr. & $\Phi(k, \cdot, m)$ incr.

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$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$.

Convexity is essential. (Nearly necessary.)

Example: $\Phi(k, \ell, m) = k + \phi(\ell)$, where ϕ str. incr.

ϕ convex: prop'n applies.

ϕ concave: $\exists\ U_S,\ U_R$, & U_R-optimal $A(\cdot)$ such that
$m \mapsto U_S(A(m), m)$ is coarsely more convex than
$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$.

(back to slide 29)

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