The comparative statics of persuasion

Gregorio Curello
University of Bonn

Ludvig Sinander
University of Oxford

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Motivation

Canonical persuasion model (Kamenica & Gentzkow, 2011)

– important model of strategic info-provision

$\hookrightarrow$ arguably most important new theory in last 15–20 years

– question: what will and won't be disclosed?

model: a <u>sender</u> designs <u>signal</u>.

(no functional-form restrictions)

– beginning to shape empirical research

e.g. Vatter (2022), Decker (2022), Crépon, Frot & Gaillac (in progress)

– more applic'ns: grades

labelling (food labels, energy ratings, ...)

credit scores

...

Motivation

Canonical persuasion model (Kamenica & Gentzkow, 2011)

Main question: 'what are optimal signals like?' Hard.

e.g. Kolotilin (2014, 2018), Gentzkow and Kamenica (2016), Dworczak and Martini (2019), Kleiner, Moldovanu and Strack (2021), Arieli, Babichenko, Smorodinsky and Yamashita (2023)

Open question: 'how do optimal signals vary with primitives?'

This paper: answer that question.

Overview

Question: when does a shift of model parameters cause sender to choose a more informative signal?

Answer: identify the necessary & sufficient conditions.

On one hand: conditions are strong.

$\implies$ often cannot draw comparative-statics conclusions.

On other hand: conditions hold in several applications.

$\hookrightarrow$ special case: 'S'-shaped payoffs (common in recent lit).

$\hookrightarrow$ special^2 cases: known comparative-statics results

(Kolotilin, Mylovanov and Zapechelnyuk, 2022; Gitmez and Molavi, 2023)

Plan

The persuasion model

‘Non-decreasing’ comparative statics

‘Increasing’ comparative statics

The persuasion model

Terminology: 'distribution' means CDF $[0,1] \to [0,1]$.

State (a bounded RV, wlog $\in [0,1]$) $\sim F_0$ ('the prior').

Sender chooses signal. (RV jointly distributed with state.)

Prior + signal + signal realisation
$\Longrightarrow$ posterior belief about state, with some mean.

Hence prior + signal $\Longrightarrow$ random posterior mean (a RV).

Assumption: sender cares only about posterior mean.
Payoff $u(m)$ from posterior mean $m \in [0,1]$.

$\hookrightarrow$ motivated by applications; common in recent lit.

Sender chooses signal to max $\mathbf{E}\big[u(\text{random posterior mean})\big]$.

Interpretation

$u(\cdot)$ is a reduced-form object.

Captures (expected) payoff from downstream interaction.

$\hookrightarrow$ e.g. actions taken by some 'receivers'.

Our analysis is robust to downstream details:
identifies <u>necessary & sufficient</u> conditions directly on u.

$\hookrightarrow$ can then check these in applications.

Application: privately informed receiver

Model of Kolotilin, Mylovanov, Zapechelnyuk and Li (2017):

Receiver chooses whether to 'participate'; sender hopes yes.

$\hookrightarrow$ example: whether to buy sender's good.

Sender provides info about value of particip'n (=state).

Outside option worth $R \sim G$, privately observed by receiver.

$$\Longrightarrow \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Question: what shifts of G cause more info-provision?

(details: slide 32)

Kolotilin's (2014) reformulation

Model: $$\max_{S \in \{\text{signals}\}} \mathbf{E}_S\big[u(\text{random posterior mean})\big] = \int u \, \mathrm{d}F_S$$

where (random posterior mean induced by S) $\sim F_S$.

Reformulation: sender chooses F_S directly.

Optimal choices: $$\operatorname*{arg\,max}_{F \text{ feasible given } F_0} \int u \, \mathrm{d}F$$

where ‘F feasible given F_0’ $\overset{\text{def'n}}{\iff}$ $\exists$ signal S such that $F_S = F$.

Fact: F feasible given F_0

$\iff$ F a mean-preserving contraction of F_0

$$\left(\overset{\text{def'n}}{\iff} \quad \int_0^x F \le \int_0^x F_0 \quad \forall x \in [0,1) \quad \& \quad \int_0^1 F = \int_0^1 F_0 \right).$$

Informativeness

Definition: F is <u>less informative</u> than G
iff $\int \psi \mathrm{d}F \leq \int \psi \mathrm{d}G$ for every convex $\psi : [0,1] \to \mathbf{R}$.

In the spirit of D. Blackwell.

Fact: F less informative than G
$\iff$ F a mean-preserving contraction of G.

'Less informative' is demanding:

frequently F is not less informative than G <u>and</u>
G is not less informative than F.

More comparisons

$$F \ \underline{\text{strictly less informative}} \text{ than } G$$
$$\overset{\text{def'n}}{\iff} \quad F \text{ less informative than } G \quad \& \quad F \neq G.$$

$$G \ \underline{\text{(str.) more informative}} \text{ than } F$$
$$\overset{\text{def'n}}{\iff} \quad F \ \text{(str.) less informative than } G.$$

In principle, argmax can have ≥ 2 elements

$\implies$ must compare <u>sets</u> of dist'ns.

This talk: assume all argmaxes singleton.

‘Increasing’ comparative statics

Question : for interim payoffs $u, v : [0,1] \to \mathbf{R}$,
what must we assume to conclude that

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \qquad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \qquad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F$$

whatever the prior F_0?

‘Non-decreasing’ comparative statics

‘Increasing’ is a lot to ask. Begin with <u>non-decreasing:</u>

Question′: for interim payoffs $u, v : [0,1] \to \mathbf{R}$,
what must we assume to conclude that

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{array}{c}\text{is } \underline{\text{not str. more}} \\ \text{info'tive than}\end{array} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F$$

whatever the prior F_0?

Plan

The persuasion model

‘Non-decreasing’ comparative statics

‘Increasing’ comparative statics

Coarse comparative convexity

Definition: for $u, v : [0,1] \to \mathbf{R}$,

u is coarsely less convex than v iff

for any $x < y$ in $[0,1]$ such that

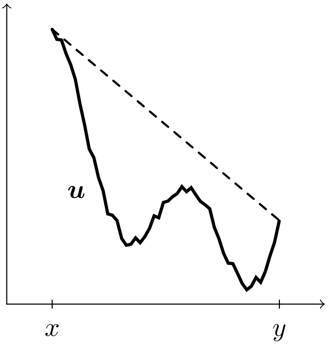
$u(\alpha x + (1-\alpha) y) \leq \alpha u(x) + (1-\alpha) u(y)$

holds $\forall\, \alpha \in (0,1)$,

$v(\alpha x + (1-\alpha) y) \leq \alpha v(x) + (1-\alpha) v(y)$

also holds $\forall\, \alpha \in (0,1)$,

and for each α, former ineq. strict $\implies$ latter ineq. strict.



Sufficient conditions

Lemma: if $v(x) = \Phi(u(x), x) \quad \forall x$
where Φ convex & $\Phi(\cdot, x)$ str. incr. $\forall x$,
then u is coarsely less convex than v.

Proof: $u(\alpha x + (1-\alpha)y) \leq(<) \alpha u(x) + (1-\alpha)u(y) \qquad \Longrightarrow$

$$v(\alpha x + (1-\alpha)y) \leq(<) \Phi\Big(\alpha u(x) + (1-\alpha)u(y), \alpha x + (1-\alpha)y\Big)$$
$$\leq \alpha v(x) + (1-\alpha)v(y)$$

by str. monotonicity & convexity. ■

Sufficient conditions

Lemma: if $v(x) = \Phi(u(x), x) \quad \forall x$
where Φ convex & $\Phi(\cdot, x)$ str. incr. $\forall x$,
then u is coarsely less convex than v.

Special case:
(usual 'less convex than')

$v = \phi \circ u$ for a convex & str. incr. $\phi : \mathbf{R} \to \mathbf{R}$

$$\left(\iff \begin{array}{l} u'' \cdot |v'| \leq v'' \cdot |u'| \\ \text{if } u, v \text{ are } C^2 \end{array} \right)$$

$\hookrightarrow$ take $\Phi(k, x) = \phi(k)$.

Special case:
(from costly info acq. lit)

$v = u + \psi$ for a convex $\psi : [0, 1] \to \mathbf{R}$

$$\left(\iff \begin{array}{l} u'' \leq v'' \\ \text{if } u, v \text{ are } C^2 \end{array} \right)$$

$\hookrightarrow$ take $\Phi(k, x) = k + \psi(x)$.

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

G improves in MLR sense   e.g. $\mu \nearrow$ if $G = N(\mu, \sigma^2)$

$$\overset{\text{def'n}}{\iff} \quad g'/g \quad \nearrow \text{ pointwise}$$

$$\iff \quad G''/G' \quad \nearrow \text{ pointwise}$$

$$\iff \quad G \text{ becomes more convex (in usual sense).}$$

So by Lemma, improved outside-option dist'n G
$\implies$ coarsely more convex u.

‘Non-decreasing’ comparative statics

Theorem 1: For upper semi-continuous $u, v : [0,1] \to \mathbf{R}$, the following are equivalent:

– u is coarsely less convex than v.

– For any prior dist’n F_0,

$$\mathop{\arg\max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{array}{c}\text{is not str. more}\\ \text{info'tive than}\end{array} \quad \mathop{\arg\max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F.$$

Proof idea

Th'm 1: For usc u & v, u is coarsely less convex than v iff

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{matrix}\text{is not str. more}\\ \text{info'tive than}\end{matrix} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F \qquad \forall F_0.$$

Necessity of 'u coarsely less convex than v': straightforward.

Sufficiency: u coarsely less convex than v

$\Longrightarrow$ $U(F) := \int u \mathrm{d}F$ interval-dominated by $V(F) := \int v \mathrm{d}F$

$$\Longrightarrow \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} U(F) \quad \begin{matrix}\text{is not str. more}\\ \text{info'tive than}\end{matrix} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} V(F)$$

1st implication: non-trivial.

2nd implication: a theorem of Quah and Strulovici (2009, 2007).

Plan

The persuasion model

'Non-decreasing' comparative statics

'Increasing' comparative statics

Halfway there

By Theorem 1, 'more convexity' is necessary & <u>not</u> sufficient for <u>increasing</u> comparative statics.

What can go wrong? Example at end of talk (if time allows).

Remaining question: what further restriction on u is needed?

Regularity

From now on, focus on <u>regular</u> u.

'Regular': slightly weaker than twice contin'sly differentiable.

(def'n: slide 33)

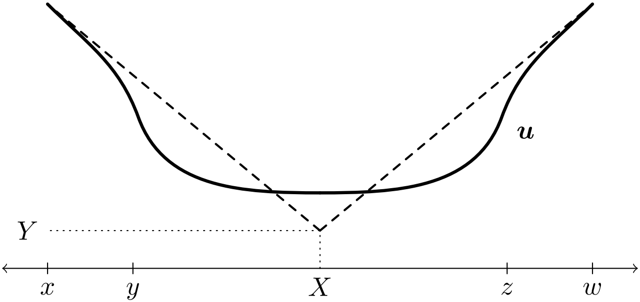
Crater property

Definition: regular $u : [0,1] \to \mathbf{R}$ sat's the crater property iff

$$\forall \;\; x < y < z < w \quad \text{s.t.} \quad u \begin{cases} \text{concave} & \text{on } [x,y] \\ \text{str. convex} & \text{on } [y,z] \\ \text{concave} & \text{on } [z,w], \end{cases}$$

have $u'(x) \neq u'(w)$, & tangents at x & at w cross at (X, Y)

s.t. (i) $y \leq X \leq z$ & (ii) $Y \leq u(X)$.



(satisfied)

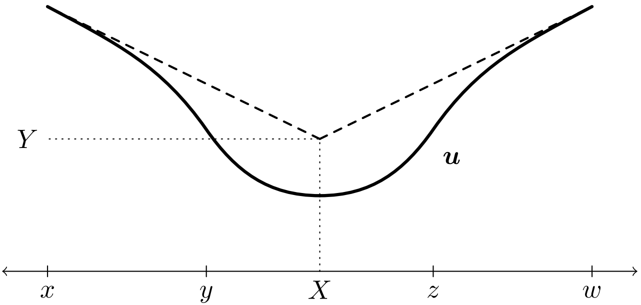
Crater property

Definition: regular $u : [0,1] \to \mathbf{R}$ sat's the crater property iff

$$\forall \;\; x < y < z < w \quad \text{s.t.} \quad u \begin{cases} \text{concave} & \text{on } [x,y] \\ \text{str. convex} & \text{on } [y,z] \\ \text{concave} & \text{on } [z,w], \end{cases}$$

have $u'(x) \neq u'(w)$, & tangents at x & at w cross at (X,Y)

s.t. (i) $y \leq X \leq z$ & (ii) $Y \leq u(X)$.

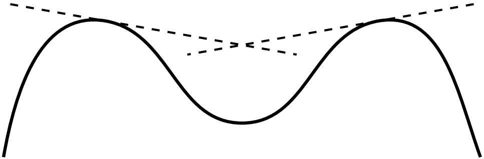


(violated)

When does the crater property hold?

Crater property is strong.

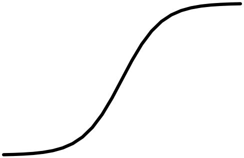
$\hookrightarrow$ e.g. rules out multiple interior local maxima.

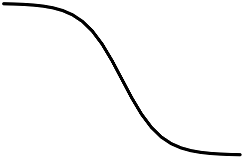


When does the crater property hold?

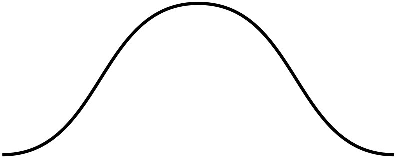
Sufficient conditions:

– 'S' shape: str. convex–concave or concave–str. convex.





– 'bell' shape: str. convex–concave–str. convex.



Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

G unimodal e.g. $G = N(\mu, \sigma^2)$

$$\overset{\text{def'n}}{\iff} \quad g \begin{Bmatrix} \text{str. incr.} & \text{on } [0, x] \\ \text{str. decr.} & \text{on } [x, 1] \end{Bmatrix} \quad \text{for some } x$$

$$\iff \quad G \begin{Bmatrix} \text{str. convex} & \text{on } [0, x] \\ \text{str. concave} & \text{on } [x, 1] \end{Bmatrix} \quad \text{for some } x$$

$\implies$ u S-shaped $\implies$ u obeys crater property.

'Increasing' comparative statics

Theorem 2: For a regular $u : [0,1] \to \mathbf{R}$, the following are equivalent:

- u satisfies the crater property.
- For every regular & coarsely more convex $v : [0,1] \to \mathbf{R}$ and every atomless convex-support F_0,

$$\operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \quad \operatorname*{arg\,max}_{F \text{ feas. given } F_0} \int v \mathrm{d}F.$$

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\implies$ u S-shaped
$\implies$ u obeys crater property.

Recall: G improves in MLR sense
$\iff$ G becomes more convex (in usual sense)
$\implies$ u becomes coarsely more convex.

By Th'm 2, G unimodal & improves in MLR sense
$\implies$ sender provides more info ($\forall$ prior).

$\hookrightarrow$ recovers Prop 1 in Kolotilin, Mylovanov and Zapechelnyuk (2022)

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\implies$ u S-shaped
$\implies$ u obeys crater property.

More generally, if G improves. E.g. $g' \nearrow$ pointwise
$\iff$ $G'' \nearrow$ pointwise
$\implies$ u coarsely more c'vex.

Application: privately informed receiver

Recall: outside option $R \sim G$, density g,
receiver participates iff $R \leq$ (posterior mean)

$$\implies \quad u(m) = \mathbf{P}(R \leq m) = G(m).$$

Recall: G unimodal $\implies$ u S-shaped
$\implies$ u obeys crater property.

Alternatively: if G becomes 'more diffuse' in sense that
g becomes less convex (in usual sense).

$\hookrightarrow$ generalises Gitmez and Molavi (2023),
who assume binary prior

Proof of sufficiency

Th'm 2: A regular u obeys crater property iff

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F$$

$\forall$ regular coarsely more c'vex v, $\forall$ atomless c'vex-supp't F_0.

Bespoke argument, relies on persuasion structure.

$\hookrightarrow$ study the dual (Dworczak & Martini, 2019)

<u>Cannot</u> use general comparative-statics results:

they require $U(F) = \int u \mathrm{d}F$ <u>(interval-)quasi-supermodular</u>

which is super-strong (requires u concave or u str. convex)

(sketch proof of necessity: slide 34)

Robustness & extensions

– restricted classes of priors F_0 (slide 35)

– 'decreasing' comparative statics (slide 37)

– constrained persuasion (slide 38)

– shifts of the prior F_0 (slide 39)

Application: alignment of interests

Question: alignment $\nearrow$ $\implies$ info-provision $\nearrow$?

Answer: yes if control convexity, no otherwise.

Setting: actions $a \in \mathcal{A}$, payoffs $U_S(a,m)$, $U_R(a,m)$,
choice $A(m)$ U_R-optimal $\Big(\in \arg\max_{a\in\mathcal{A}} U_R(a,m)\Big)$
$\implies u(m) = U_S\big(A(m), m\big)$.

Example: shift from $(a,m) \mapsto U_S(a,m)$
to $(a,m) \mapsto U_S(a,m) + \phi\Big(U_R(a,m)\Big)$
where ϕ str. incr. (' alignment $\nearrow$ ')

ϕ convex: u becomes coarsely more convex.
$\forall\ U_S,\ U_R,\ \ \&\ \ U_R$-optimal $A(\cdot)$

(general: slide 40)

ϕ concave: u may become coarsely less convex!
$\exists\ U_S,\ U_R,\ \ \&\ \ U_R$-optimal $A(\cdot)$

Conclusion

Open question in canonical persuasion model:

when does a shift of model parameters cause
sender to choose a <u>more informative</u> signal?

Complete answer:

u obeys crater property + becomes coarsely more convex.

Applied upshot:

– easy-to-check sufficient conditions

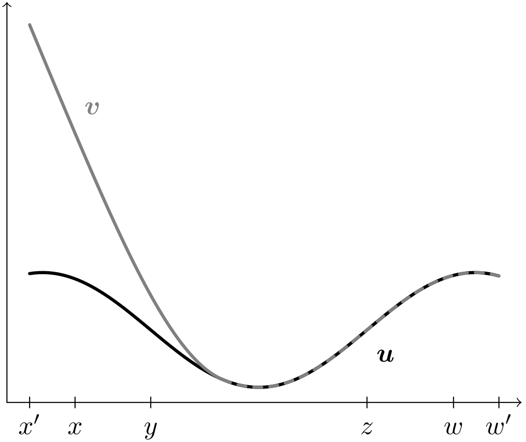
– applications (see paper)

Conclusion

Remaining questions:

- further applications
- case when ≥ 2 moments matter (not just mean).

Thanks!



Application: details

Detail 1: assume $R \perp\!\!\!\perp$ (value of particip'n).

Detail 2: Can sender do better by offering a <u>menu</u> of signals?
No. (Kolotilin, Mylovanov, Zapechelnyuk & Li, 2017, Th'm 1)

(back to slide 7)

Regularity: definition

Definition: $u : [0,1] \to \mathbf{R}$ is regular iff both

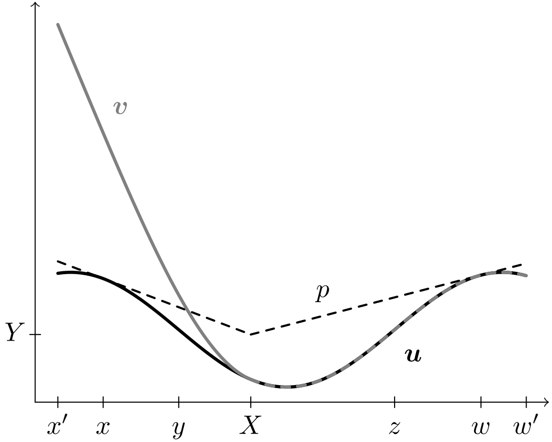
(i) u is contin's & possesses contin's & bounded derivative $u' : (0,1) \to \mathbf{R}$

(ii) $[0,1]$ may be partitioned into finitely many intervals on which u is either affine, str. convex, or str. concave.

Sufficient condition: u twice contin'sly differentiable.

(back to slide 21)

Sketch proof of necessity



Suppose u regular & violates crater.

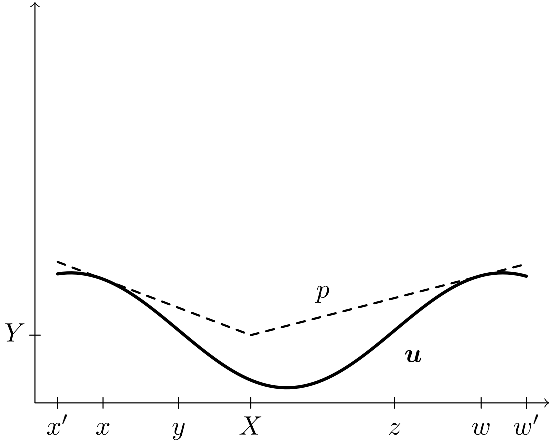
Construct F_0:

- atomless
- support $[x', w']$
- $\frac{\int_0^X \xi F_0(\mathrm{d}\xi)}{F_0(X)} = x$
- $\frac{\int_X^1 \xi F_0(\mathrm{d}\xi)}{1-F_0(X)} = w.$

Construct v:

- on $[0, X]$, $\geq u$ & str. convex
- on $[X, 1]$, $= u$.

Sketch proof of necessity



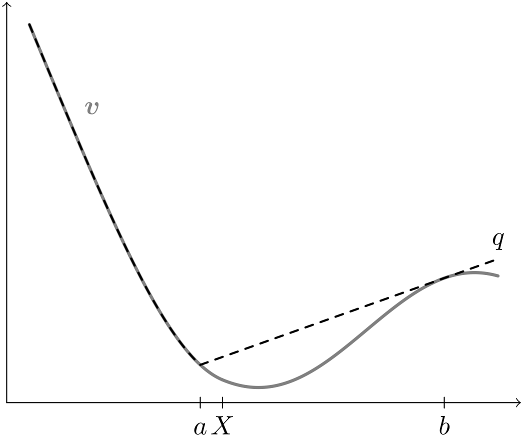
For u, optimal dist'n F reveals (only) whether state $\gtrless X$:

no pooling acr. X.

Proof: $\forall\, H$ less info. than F_0,

$$
\begin{aligned}
& \int u \mathrm{d}F \\
&= \int p \mathrm{d}F && u \stackrel{F\text{-a.e.}}{=} p \\
&= \int p \mathrm{d}F_0 && p \text{ aff. } [0, X] \ \& \ [X, 1] \\
&\geq \int p \mathrm{d}H && p \text{ convex} \\
&\geq \int u \mathrm{d}H && p \geq u. \quad \blacksquare
\end{aligned}
$$

Sketch proof of necessity



where $\quad b = \dfrac{1}{1 - F_0(a)} \displaystyle\int_a^1 \xi F_0(\mathrm{d}\xi)$

v S-shaped $\implies$ optimal dist'n G reveals $[0, a)$, pools $[a, 1]$:

so pools across X.

Proof: $\forall\, H$ less info. than F_0,

$$
\begin{aligned}
& \textstyle\int v \mathrm{d}G \\
&= \textstyle\int q \mathrm{d}G && v \overset{G\text{-a.e.}}{=} q \\
&= \textstyle\int q \mathrm{d}F_0 && q \text{ aff. } [a, 1] \\
&\geq \textstyle\int q \mathrm{d}H && q \text{ convex} \\
&\geq \textstyle\int v \mathrm{d}H && q \geq v. \quad \blacksquare
\end{aligned}
$$

(back to slide 27)

Restricted classes of priors

Th'm 2: crater property necessary if consider all priors F_0.

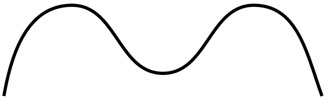
Robustness: necessary even if consider only a single F_0:

Prop'n: Provided $|\operatorname{supp} F_0| \geq 3$, $\exists$ regular $u, v : [0,1] \to \mathbf{R}$ such that u is coarsely less convex than v, but

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{matrix}\text{is not less} \\ \text{info'tive than}\end{matrix} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F.$$

Can choose u M-shaped & v S-shaped.

'M-shaped' = concave–str. convex–concave.



($|\operatorname{supp} F_0| \leq 2$: next slide) (back to slide 28)

Binary priors

Binary prior: F_0 with $|\operatorname{supp} F_0| \leq 2$.

Effectively: state is binary.

Binary priors are special—no need for crater property:

Prop'n: For upper semi-continuous $u, v : [0,1] \to \mathbf{R}$, the following are equivalent:

- u is coarsely less convex than v.
- For any binary prior dist'n F_0,

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F.$$

(back to slide 28)

'Decreasing' comparative statics

Symmetric counterpart to question answered by Th'm 2:

what ass'ns on v ensure comparative statics

with any coarsely <u>less</u> convex u, whatever the prior F_0?

Answer: need super-strong ass'ns:

Prop'n: For a regular $v : [0,1] \to \mathbf{R}$,
the following are equivalent:

- v is either <u>concave</u> or <u>str. convex</u>.
- For every regular & coarsely less convex $u : [0,1] \to \mathbf{R}$ and every atomless convex-support F_0,

$$\underset{F \text{ feas. given } F_0}{\arg\max} \int u \mathrm{d}F \quad \begin{array}{c}\text{is less}\\ \text{info'tive than}\end{array} \quad \underset{F \text{ feas. given } F_0}{\arg\max} \int v \mathrm{d}F.$$

(back to slide 28)

Constrained persuasion

Sender may face constraints on choice of signal. Growing lit.

Two natural constraints:

– only monotone partitional signals

– only signals that send $\leq K$ messages, for some $K \in \mathbf{N}$

Prop'n: in both cases, crater property remains necessary.

(back to slide 28)

Shifts of the prior

Shifts of prior F_0 instead of payoff u.

Interpret'n: change in info available to sender.

Prop'n: there are no $F_0 \neq G_0$ such that

$$\mathop{\arg\max}_{F \text{ feas. given } F_0} \int u \mathrm{d}F \quad \begin{matrix}\text{is less}\\ \text{info'tive than}\end{matrix} \quad \mathop{\arg\max}_{F \text{ feas. given } G_0} \int u \mathrm{d}F$$

for every regular and S-shaped $u : [0,1] \to \mathbf{R}$.

Upshot: comparative statics highly u-sensitive.
No result across all u, not even all S-shaped u.

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Application: alignment of interests, in general

Alignment $\nearrow$: shift from $(a, m) \mapsto U_S(a, m)$

to $(a, m) \mapsto \Phi\Big(U_S(a, m), U_R(a, m), m\Big)$

where Φ an alignment-incr'ing utility transform'n (AIUT):

- <u>utility transformation:</u> $\Phi(\cdot, \ell, m)$ str. incr. $\forall\, \ell, m$
- <u>alignment-increasing:</u> $\Phi(k, \cdot, m)$ incr. $\forall\, k, m.$

Prop'n: For any <u>convex</u> AIUT Φ,

$m \mapsto U_S(A(m), m)$ is coarsely less convex than

$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$

$\forall\ U_S,\ U_R, \quad \forall\ U_R$-optimal $A(\cdot)$.

Application: alignment of interests, in general

AIUT: Φ such that $\Phi(\cdot, \ell, m)$ str. incr. & $\Phi(k, \cdot, m)$ incr.

Prop'n: $\forall$ <u>convex</u> AIUT Φ, $\forall\ U_S, U_R$, $\forall\ U_R$-optimal $A(\cdot)$,
$m \mapsto U_S(A(m), m)$ is coarsely less convex than
$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$.

Convexity is essential. (Nearly necessary.)

Example: $\Phi(k, \ell, m) = k + \phi(\ell)$, where ϕ str. incr.

ϕ convex: prop'n applies.

ϕ concave: $\exists\ U_S,\ U_R,$ & U_R-optimal $A(\cdot)$ such that
$m \mapsto U_S(A(m), m)$ is coarsely <u>more</u> convex than
$m \mapsto \Phi\Big(U_S(A(m), m), U_R(A(m), m), m\Big)$.

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