Agenda-manipulation in ranking

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Ranking by committee

Many organisations governed by committee. Typically

- committee sets *priorities*
- day-to-day decisions delegated to executives.

Simple example: hiring.

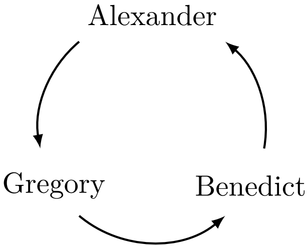
- uncertainty about which candidates would accept offer
- hiring committee ranks the candidates
- delegates task of extending offers (to 1^{st}; to 2^{nd}; etc.)

For lack of info, committee doesn't pick an alternative;

instead *ranks* the alternatives.

Agenda-setting

Majority will may contain (Condorcet) cycles:

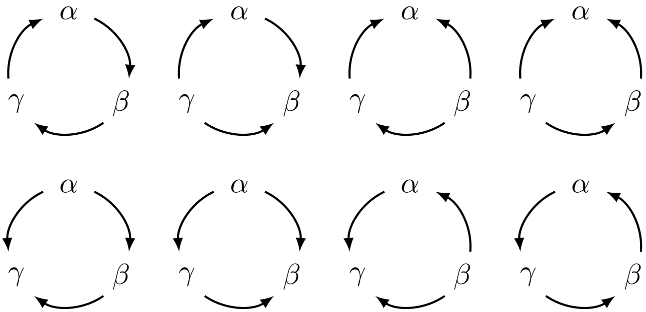


Committee's *chair* chooses order of pairwise votes.

Transitivity imposed.

Uncertainty

Chair does not know the majority will, W.



Regret-free strategies

Question: how much influence can chair exert? & how?

Answer: $\exists$ *regret-free* strategy:

reaches a ranking 'as good as' the full-info optimum,

whatever the majority will.

Seek to understand RF qualitatively:

- what do RF strategies have in common?
- what distinguishes them from each other?

Related literature

agenda-manipulation: Farquharson (1969), Black (1958), Miller (1977), Banks (1985)

$\hookrightarrow$ incomplete info: Ordeshook and Palfrey (1988), recently Moldovanu & co-authors

social choice: Zermelo (1929), Wei (1952), Kendall (1955)

$\hookrightarrow$ Copeland: Copeland (1951), Rubinstein (1980)

$\hookrightarrow$ Kemeny–Slater: Kemeny (1959), Slater (1961), Young and Levenglick (1978), Young (1986, 1988)

$\hookrightarrow$ fair-bets: Daniels (1969), Moon and Pullman (1970), Slutzki and Volij (2005)

Plan

Preliminaries

Do RF strategies exist?

What do RF strategies have in common?

How do RF strategies differ?

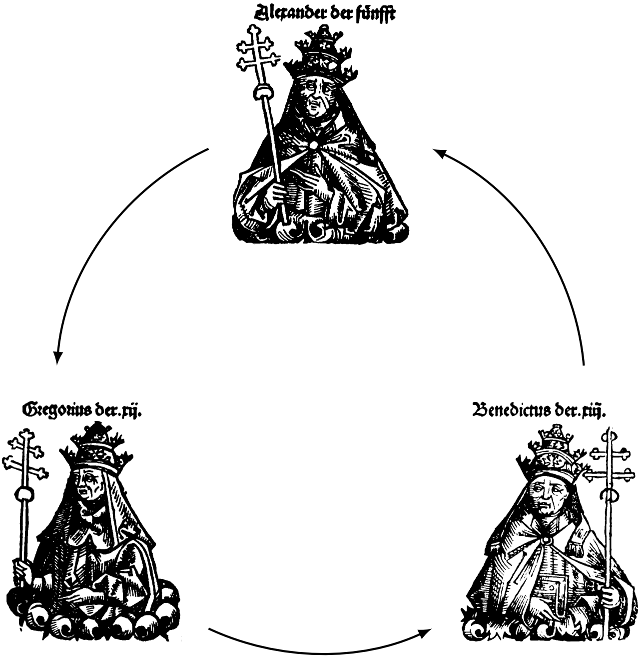
Preferences

Chair has preference $\succ$ over alternatives.

Ranking R is *more aligned with* $\succ$ than R'
iff whenever $x \succ y$ and $x \; R' \; y$, also $x \; R \; y$.

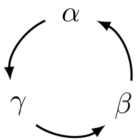
Chair prefers more aligned rankings . . . and that's all.

Hiring: more aligned $\iff$ hires $\succ$-better candidate at every realisation of uncertainty.



(anti-)popes in 1409–10, from Schedel (1493)

Example



and $\alpha \succ \beta \succ \gamma$

W-reachable rankings:
$\beta \; R \; \alpha \; R \; \gamma$, $\quad \alpha \; R' \; \gamma \; R' \; \beta \quad$ and $\quad \gamma \; R'' \; \beta \; R'' \; \alpha$.

R and R' are more aligned with $\succ$ than R''
and are incomparable to each other.

$\implies$ R and R' cannot be W-feasibly improved upon.

Regret-free strategies

A ranking is W*-unimprovable* iff no other ranking is both

(i) reachable under W and

(ii) more aligned with $\succ$.

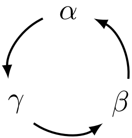
With perfect knowledge of W,
W-unimprovability is the strongest optimality concept.

A *regret-free* strategy
reaches a W-unimprovable ranking under every W.

Efficiency

A W-*efficient* ranking
is one that ranks x above y whenever both $x \succ y$ and $x \, W \, y$.

Example:



and $\alpha \succ \beta \succ \gamma$

W-efficient rankings: $\succ$ itself, $\beta \, R \, \alpha \, R \, \gamma$ and $\alpha \, R' \, \gamma \, R' \, \beta$.

Definition.
A strategy is *efficient* iff under any majority will W,
it reaches a W-efficient ranking.

W-efficiency implies W-unimprovability

Lemma 1.
For any majority will W,
a W-efficient ranking is W-unimprovable.

Corollary.
Any efficient strategy is regret-free.

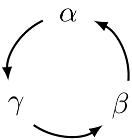
Intuition for Lemma 1

Given W, call a pair $x \succ y$ $\begin{cases} \text{an } \textit{agreement pair} & \text{if } x\, W\, y \\ \text{a } \textit{disagreement pair} & \text{if } y\, W\, x. \end{cases}$

Efficiency: rank every agreement pair 'right'.

Disagreement pairs can be ranked 'right' only via transitivity.

Example:



and $\alpha \succ \beta \succ \gamma$

W-efficient $\beta\, R\, \alpha\, R\, \gamma$: $\begin{cases} \alpha, \beta & \text{voted on} \implies \text{ranked 'wrong'} \\ \beta, \gamma & \text{not voted on; ranked 'right'.} \end{cases}$

To improve, must refrain from vote on α, β
$\implies$ vote on β, γ $\implies$ rank this pair 'wrong'.

Proof of Lemma 1

Fix W, W-efficient R, and W-reachable $R' \neq R$.
We'll show that R' is not MAW $\succ$ than R.

Since $R' \neq R$, $\exists$ alternatives x, y such that $x \; R' \; y$ and $y \; R \; x$.
Enumerate alternatives that R' ranks between x and y as

$$x = z_1 \; R' \; z_2 \; R' \cdots R' \; z_N = y.$$

Since R' is W-reachable, must have $z_1 \; W \; z_2 \; W \cdots W \; z_N$.

There has to be $n < N$ at which $z_{n+1} \; R \; z_n$,
else we'd have $x \; R \; y$ by transitivity of R.

It must be that $z_{n+1} \succ z_n$,
else we'd have $z_n \; R \; z_{n+1}$ by $z_n \; W \; z_{n+1}$ and W-efficiency of R.

So (z_n, z_{n+1}) is ranked 'right' by R and 'wrong' by R'
$\implies$ R' is not MAW $\succ$ than R. ∎

Plan

Preliminaries

Do RF strategies exist?

What do RF strategies have in common?

How do RF strategies differ?

Insertion sort

Label the alternatives $\mathcal{X} \equiv \{1, \ldots, n\}$ so that $1 \succ \cdots \succ n$.

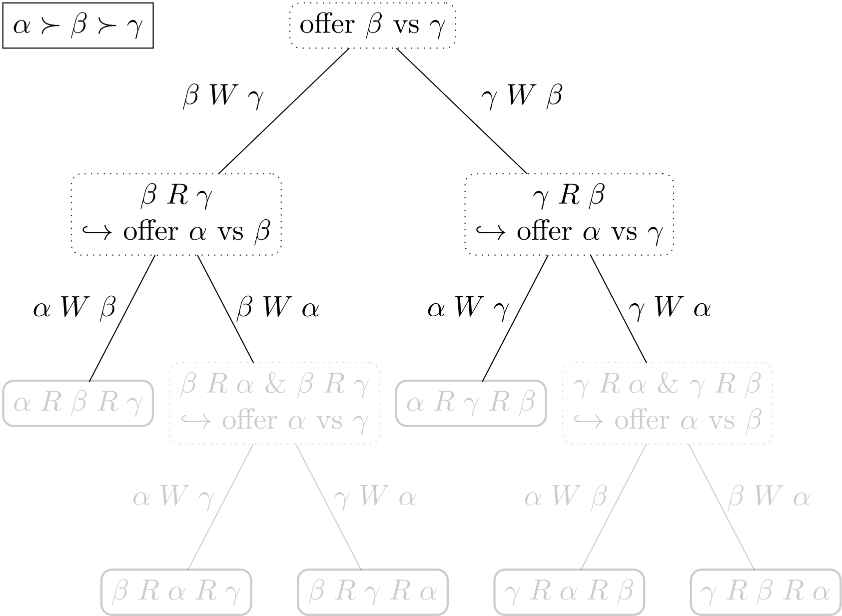
Insertion sort strategy: for each $k \in \{n-1, \ldots, 1\}$,

– totally rank $\{k+1, \ldots, n\}$

(write $x_{k+1} \; R \cdots R \; x_n$, where $\{x_{k+1}, \ldots, x_n\} \equiv \{k+1, \ldots, n\}$)

– 'insert' k into $\{k+1, \ldots, n\}$:

pit k against the highest-ranked (x_{k+1});
then (if k lost) pit k against the 2^{nd}-highest-ranked (x_{k+2});
…



$\alpha \succ \beta \succ \gamma$
offer β vs γ
$\beta\ W\ \gamma$
$\gamma\ W\ \beta$
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Insertion sort is regret-free

Theorem 1.
The insertion-sort strategy is efficient, hence regret-free.

Proof of Theorem 1

Fix W; let R be ranking reached by IS under W.
Fix x, y with $x \succ y$ and $x \; W \; y$; must show that $x \; R \; y$.

Enumerate all alternatives $\succ$-worse than x as $z_1 \; R \cdots R \; z_K$.
Note that $z_k = y$ for some $k \leq K$.

By definition of IS,
x is pitted against $z_1, z_2, \ldots$ in turn until it wins a vote.

- if x loses against $z_1, \ldots, z_{k-1}$,
  then it is pitted against $z_k = y$ and wins (since $x \; W \; y$)
  $\implies x \; R \; y$.

- if x wins against z_ℓ for $\ell < k$,
  then $x \; R \; z_\ell \; R \cdots R \; z_k = y$
  $\implies x \; R \; y$ (by transitivity of R). ■

Plan

Preliminaries

Do RF strategies exist?

What do RF strategies have in common?

How do RF strategies differ?

What (other) strategies are regret-free?

We've shown that RF strategies exist.

What do RF strategies have in common?

$\iff$ qualitatively, what does RF-ness require?

Characterisation of outcomes

Recall that W-efficiency $\Longrightarrow$ W-unimprovability (Lemma 1).

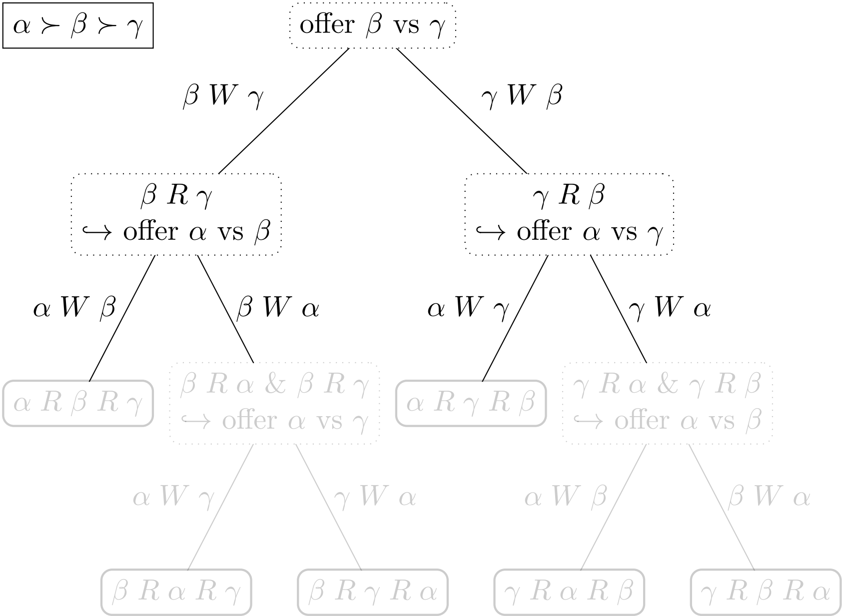
The converse is false:
a W-unimprovable ranking need not be W-efficient.

(counter-example: slide 38)

But only efficiency ensures unimprovability robustly across Ws:

Theorem 2.
A strategy is regret-free iff it is efficient.

(tightness: slide 40)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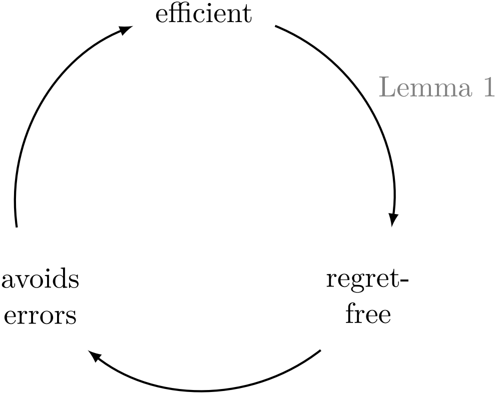
Characterisation of behaviour

Theorem 3.
A strategy is regret-free iff
it never misses an opportunity or takes a risk.

(formal definitions: slide 41) (tightness: slide 42)

Proof of Theorems 2 & 3



(details: slide 43)

Plan

Preliminaries

Do RF strategies exist?

What do RF strategies have in common?

How do RF strategies differ?

Reverse insertion sort

Label the alternatives $\mathcal{X} \equiv \{1, \ldots, n\}$ so that $1 \succ \cdots \succ n$.

Reverse insertion sort strategy: for each $k \in \{2, \ldots, n\}$,

– totally rank $\{1, \ldots, k-1\}$

(write $x_1 \; R \cdots R \; x_{k-1}$, where $\{x_1, \ldots, x_{k-1}\} \equiv \{1, \ldots, k-1\}$)

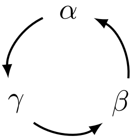
– 'insert' k into $\{1, \ldots, k-1\}$:

pit k against the lowest-ranked (x_{k-1});
then (if k won) pit k against the 2$^{\text{nd}}$-lowest-ranked (x_{k-2});
...

Reverse IS is efficient (by Theorem-1 argument)
$\implies$ regret-free. (by Lemma 1)

IS vs. reverse IS

Example:



and $\alpha \succ \beta \succ \gamma$

W-reachable, W-efficient rankings:

$\beta \; R \; \alpha \; R \; \gamma$ and $\alpha \; R' \; \gamma \; R' \; \beta$.

Reverse insertion sort reaches R. Insertion sort reaches R'.

Prioritisation: 'right' ranking of $\begin{cases} \beta, \gamma & \text{for reverse IS} \\ \alpha, \beta & \text{for IS.} \end{cases}$

Prioritisation

Every RF strategy ranks agreement pairs 'right'. (Theorem 2)

$(x \succ y \;\; \& \;\; x \, W \, y)$

Disagreement pairs:

- some ranked by vote $\implies$ bad.
- others by impositions of transitivity
  $\hookrightarrow$ *favourable* ones! (Theorem 3)
  $\implies$ good.
- trade-off: to rank one pair by transitivity,
  must offer votes on others.

$\implies$ RF strategies differ in *which*
favourable impositions of transitivity they exploit.

How does IS prioritise?

Label the alternatives $\mathcal{X} \equiv \{1, \ldots, n\}$ so that $1 \succ \cdots \succ n$.

IS leaves 1 for last: ranks $\{2, \ldots, n\}$, then 'inserts' 1.

$\hookrightarrow$ maximises favourable impositions of transitivity involving 1.

Subject to that, IS leaves 2 for last.
Subject to *that*, IS leaves 3 for last. etc.

Suggests lexicographic prioritisation:
among all strategies, IS optimises position of 1;
among such strategies, it optimises position of 2; etc.

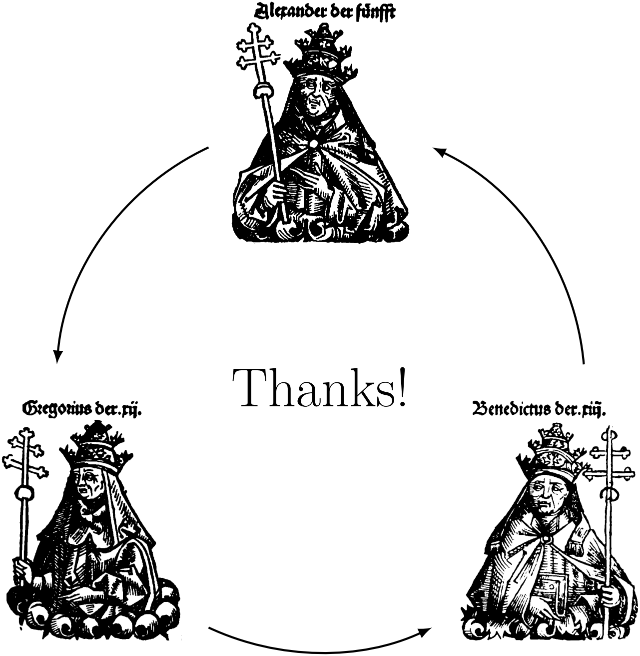
Lexicographic prioritisation

For alternative x, strategy σ and majority will W,
write $R^\sigma(W)$ for ranking reached under σ and W, and

$$N_x^\sigma(W) \coloneqq |\{y \in \mathcal{X} : x \succ y \text{ and } x \; R^\sigma(W) \; y\}|.$$

Definition.
Given $x \in \mathcal{X}$, σ is *better for x than* σ' iff
$|\{W : N_x^\sigma(W) \geq k\}| \geq |\{W : N_x^{\sigma'}(W) \geq k\}| \quad \forall k \in \{1, \ldots, n{-}1\}.$
If $\sigma \in \Sigma$ is better for x than each $\in \Sigma$, it is *best for x among* Σ.

Theorem 4.
A strategy is outcome-equivalent to insertion sort iff
among all strategies, it is best for 1;
among such strategies, it is best for 2; and so on.



Thanks!

(anti-)popes in 1409–10, from Schedel (1493)

Interaction

Write R_t for what has been decided by the end of period t.
(A *proto-ranking:* an irreflexive, ~~total~~ & transitive relation on $\mathcal{X}$.)

Initially, nothing is decided: $R_0 = \varnothing$.

In each period t, unless R_{t-1} is already total,

- chair offers vote on an unranked (by R_{t-1}) pair $x, y \in \mathcal{X}$
- each voter $i \in \{1, \ldots, I\}$ votes for either x or y
- winner is ranked above loser, and transitivity is imposed:

$$R_t = \text{transitive closure of } \begin{cases} R_{t-1} \cup \{(x,y)\} & \text{if } x \text{ won} \\ R_{t-1} \cup \{(y,x)\} & \text{if } y \text{ won.} \end{cases}$$

(back to slide 3)

Why this protocol?

Our *transitive protocol* denies the chair arbitrary power:

- *committee sovereignty:*
  if x beats y in a vote, then x is ranked above y.

- *democratic legitimacy:*
  enough votes must be offered that
  every pair is linked by a chain of majorities.

Any protocol that denies the chair arbitrary power
is exactly the transitive protocol
with restrictions on which unranked pairs the chair can offer.

(back to slide 3)

A characterisation of our protocol

A *ballot* is a set $B \subseteq \mathcal{X}$ of ≥ 2 alternatives.
An *election* is (B, V) where B is a ballot and $V : \{1, \ldots, I\} \to B$.
A *history* is a sequence of elections with distinct ballots.

Write $h \sqsubseteq h'$ iff h is a truncation of h'. For a set $\mathcal{H}$ of histories, write $h \in \tau(\mathcal{H})$ ('h is terminal') iff $h \in \mathcal{H}$ and there is no $h' \sqsupset h$ in $\mathcal{H}$.

A *protocol* is a set $\mathcal{H}$ of ('permitted') histories s.t.

- $h \sqsubseteq h' \in \mathcal{H}$ implies $h \in \mathcal{H}$, and
- $((B_1, V_1), \ldots, (B_t, V_t)) \in \mathcal{H}$ implies $((B_1, V_1), \ldots, (B_t, V_t')) \in \mathcal{H} \;\; \forall V_t'$

and a map ρ that assigns a ranking to each terminal $h \in \mathcal{H}$.

$(\mathcal{H}, \rho)$ is a *restriction* of $(\mathcal{H}', \rho')$ iff $\tau(\mathcal{H}) \subseteq \tau(\mathcal{H}')$ and $\rho = \rho'|_{\tau(\mathcal{H})}$.

A characterisation of our protocol

For a history $h = ((B_t, V_t))_{t=1}^{T}$,

– write $x \; S^h \; y$ iff $x, y \in B_t$ and $|\{i : V_t(i) = x\}| \geq |\{i : V_t(i) = y\}| \;\; \exists t$

– say that h *gives the committee a say on* x, y iff
$\{z_1, z_L\} = \{x, y\}$ for some sequence $z_1 \; S^h \; z_2 \; S^h \cdots S^h \; z_L$.

Proposition.

A protocol is a restriction of our transitive protocol iff it satisfies

(i) *binary ballots:* for any $((B_t, V_t))_{t=1}^{T} \in \mathcal{H}$,
we have $|B_1| = \cdots = |B_T| = 2$.

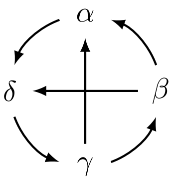
(ii) *committee sovereignty:* at any terminal $h = ((B_t, V_t))_{t=1}^{T} \in \mathcal{H}$,
if $|\{i : V_t(i) = x\}| > I/2$ and $y \in B_t$, then $x \; \rho(h) \; y$.

(iii) *democratic legitimacy:* every terminal $h \in \mathcal{H}$
gives the committee has a say on each pair of alternatives.

(back to slide 3)

Counter-example to the converse of Lemma 1

$\mathcal{X} = \{\alpha, \beta, \gamma, \delta\}$ with $\alpha \succ \beta \succ \gamma \succ \delta$ and



The ranking $\alpha \; R \; \delta \; R \; \gamma \; R \; \beta \ldots$

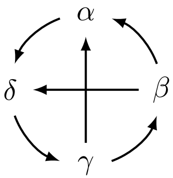
(– is W-reachable: offer $\{\alpha, \delta\}$, $\{\delta, \gamma\}$, $\{\gamma, \beta\}$.)

– is W-unimprovable,
since no other W-reachable ranking ranks α above β.
(Because there's only one directed path in W from α to β.)

– is not W-efficient, since $\delta \; R \; \beta$.

(back to slide 23)

Necessity of efficiency



and $\alpha \succ \beta \succ \gamma \succ \delta$.

non-W-efficient rankings feature sacrifices ($\delta \; R \; \beta$)

…which may pay off ($\alpha \; R \; \beta$) $\Longrightarrow$ W-unimprovable ranking

…or not $\Longrightarrow$ non-W-unimprovable ranking.

In fact, any sacrifice can fail to pay off
$\Longrightarrow$ inefficient strategies cannot be regret-free.

(back to slide 23)

Theorem 2 tightness

The characterisation in Theorem 2 is tight in the following sense:

Proposition 1.
For any majority will W and W-reachable W-efficient ranking R, some regret-free strategy reaches R under W.

Thus for every majority will W,

$$\begin{aligned} &\{R : \exists \text{ RF strategy that reaches } R \text{ under } W\} \\ = &\{R : R \text{ is } W\text{-reachable and } W\text{-efficient}\} \end{aligned}$$

($\subseteq$ by Theorem 2, $\supseteq$ by Proposition 1)

(back to slide 23)

Formal definition of errors

Definition.
Let R be an incomplete ranking, and let $x \succ y$ be unranked.
$\left(\begin{array}{l}\text{an irreflexive \&} \\ \text{transitive relation}\end{array}\right)$

(1) Offering $\{x, y\}$ for a vote *misses an opportunity (at R)* iff there is an alternative z s.t. $x \succ z \succ y$ and $y \not R\, z \not R\, x$.

(2) Offering $\{x, y\}$ for a vote *takes a risk (at R)* iff there is an alternative z s.t. either

- $z \succ y, \quad x\, R\, z \quad$ and $\quad y \not R\, z, \quad$ or
- $x \succ z, \quad z\, R\, y \quad$ and $\quad z \not R\, x.$

(back to slide 25)

Theorem 3 tightness

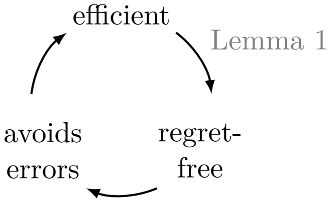
Proposition 2.

After any error-free history,
there is a pair that can be offered without committing an error.

Yields tightness:
for any W and any sequence of pairs that is error-free under W,
some regret-free strategy offers this sequence under W.

(back to slide 25)

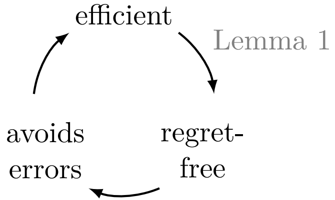
Proof of Theorems 2 & 3



Avoids errors $\implies$ efficient: contra-positive.

- suppose σ not efficient $\implies$ under some W, reach R s.t. $y \, R \, x$ despite $x \succ y$ and $x \, W \, y$.
- must be due to unfavourable imposition of transitivity.
- argue that error-avoidance precludes unfavourable impositions of transitivity.

Proof of Theorems 2 & 3



Regret-free $\implies$ no errors: contra-positive.

– suppose σ erroneously offers $x \succ y$ under some W

$$\implies \exists\, W' \quad \text{s.t.} \quad \begin{cases} y\ R\ x & \text{for } R \text{ reached by } \sigma \text{ under } W' \\ x\ R'\ y & \text{for some other } W'\text{-reachable } R'. \end{cases}$$

– carefully construct W' and R' so that

every other pair z, w ranked 'right' by R
also ranked 'right' by R'.

(back to slide 26)

The (recursive) amendment procedure

Amendment procedure: pit $n-1$ against n, then pit the winner against $n-2$, then pit the winner against $n-3$, and so on. Call the winner of the final round the *final winner.*

Recursive amendment procedure (a.k.a. 'selection sort'):

– run the AP on $\{1, \ldots, n\}$; call the final winner y_1.

– run the AP on $\{1, \ldots, n\} \setminus \{y_1\}$; call the final winner y_2.

– $\ldots$

The resulting ranking is $y_1 \; R \; y_2 \; R \cdots R \; y_{n-1} \; R \; y_n$.

Proposition 3.

Recursive amendment and insertion sort are outcome-equivalent.

(back to slide 32)

History-invariant voting

By using the majority will W, we implicitly assume (approximately) history-invariant voting.

– reasonable if voting is non-strategic or 'expressive'

– not unreasonable if voting is strategic.

Strategic voting

Each voter i has a preference $\succ_i$ over alternatives,
and prefers rankings more aligned with $\succ_i$.

A voter's *strategy* specifies how to vote at each history.
The *sincere strategy:* vote for your favourite. History-invariant!

Ranking when chair [voters] use σ [σ_i, σ_{-i}] denoted $R(\sigma, \sigma_i, \sigma_{-i})$.

Definition.
A strategy σ_i is *dominant* iff for any alternative strategy σ_i',

($\nexists$) there exists no profile σ, σ_{-i} such that $R(\sigma, \sigma_i', \sigma_{-i})$ is distinct from, and MAW $\succ_i$ than, $R(\sigma, \sigma_i, \sigma_{-i})$.

($\exists$) there exists a profile σ, σ_{-i} such that $R(\sigma, \sigma_i, \sigma_{-i})$ is distinct from, and MAW $\succ_i$ than, $R(\sigma, \sigma_i', \sigma_{-i})$.

Proposition 4.

The sincere strategy is (uniquely) dominant.

(back to slide 3)

References I

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