

AGENDA-MANIPULATION IN RANKING

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Ranking by committee

A committee must rank a set of alternatives.

Hiring:

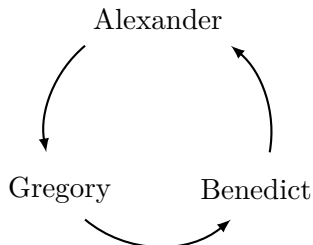
- alternatives are candidates for a job
- uncertainty about who will accept
- hiring committee decides to whom to offer the job, to whom next if the first candidate declined, etc.

Party lists:

- alternatives are a political party's parliamentary candidates
- party's leadership committee ranks them ('party list')
- the K highest-ranked candidates get parliamentary seats, where K is (uncertain) \neq seats the party wins in an election

Interaction

The majority will may contain (Condorcet) cycles:



The committee's *chair* chooses the order of pairwise votes.

Transitivity is imposed.

Preferences

The chair has a preference \succ over alternatives.

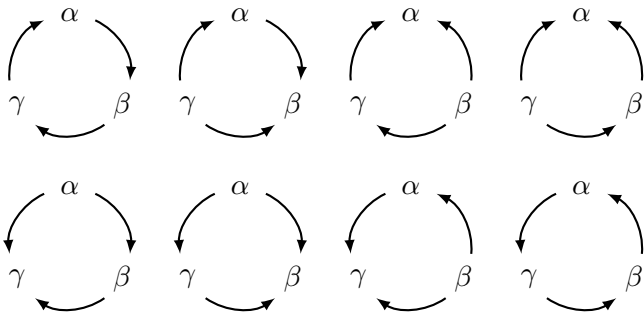
Ranking R is *more aligned with* \succ than R'
iff whenever $x \succ y$ and $x R' y$, also $x R y$.

The chair prefers rankings that are more aligned with \succ .

Hiring: a more aligned ranking is exactly one that hires a \succ -better candidate at every realisation of uncertainty.

Unknown majority will

The chair does not know the majority will, W .



Regret-free strategies

A ranking is *W-unimprovable* iff no other ranking is both

- (i) reachable under W and
- (ii) more aligned with \succ .

With perfect knowledge of W ,
 W -unimprovability is the strongest optimality concept.

A *regret-free* strategy
reaches a W -unimprovable ranking under every W .

Results

We introduce a strategy called *insertion sort*.

Theorem 1.

Insertion sort is regret-free.

What (other) strategies are regret-free?

Theorem 2: characterisation of *outcomes*.

Theorem 3: characterisation of *behaviour*.

What's special about insertion sort?

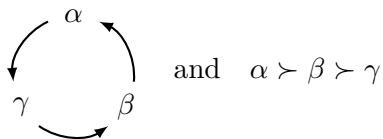
Theorem 4: IS is characterised by a lexicographic property.

Related literature

- **agenda-manipulation:** Black (1958), Farquharson (1969), Miller (1977), Banks (1985), Barberà and Gerber (2017), Roessler, Shelegia and Strulovici (2018)
 - ... with incomplete info: Ordeshook and Palfrey (1988), recent work by Benny Moldovanu & co-authors
- **social choice:** Zermelo (1929), Wei (1952), Kendall (1955)
 - Copeland's method: Copeland (1951), Rubinstein (1980)
 - Kemeny–Slater method: Kemeny (1959), Slater (1961), Young and Levenglick (1978), Young (1986, 1988)
 - fair-bets method: Daniels (1969), Moon and Pullman (1970), Slutzki and Volij (2005)

(references: slide 29)

Example



Rankings reachable under W :

$\beta R \alpha R \gamma$, $\alpha R' \gamma R' \beta$ and $\gamma R'' \beta R'' \alpha$.

R and R' are more aligned with \succ than R''
and are incomparable to each other.

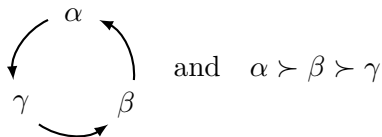
$\implies R$ and R' are W -unimprovable.

Efficiency

A W -efficient ranking

is one that ranks x above y whenever both $x \succ y$ and $x W y$.

Example.



W -efficient rankings: \succ itself, $\beta R \alpha R \gamma$ and $\alpha R' \gamma R' \beta$.

Definition.

A strategy is *efficient* iff for any majority will W , its outcome under W is W -efficient.

W -efficiency implies W -unimprovability

Lemma 1.

For any majority will W ,
a W -efficient ranking is W -unimprovable.

Corollary.

Any efficient strategy is regret-free.

Proof of Lemma 1

Fix a W , a W -efficient R , and a W -reachable $R' \neq R$.
Suppose toward a contradiction that R' is MAW \succ than R .

Since $R' \neq R$, \exists alternatives x, y such that $x R' y$ and $y R x$.
Enumerate the alternatives that R' ranks between x and y as

$$x = z_1 R' z_2 R' \cdots R' z_N = y.$$

Since R' is W -reachable, we must have $z_1 W z_2 W \cdots W z_N$.

There has to be $n < N$ at which $z_{n+1} R z_n$,
else we'd have $x R y$ by transitivity of R .

It must be that $z_{n+1} \succ z_n$,
else we'd have $z_n R z_{n+1}$ by $z_n W z_{n+1}$ and W -efficiency of R .

So (z_n, z_{n+1}) is ranked 'right' by R and 'wrong' by R'
... which is absurd since R' is MAW \succ than R . ■

Insertion sort

Label the alternatives $\{1, \dots, n\}$ so that $1 \succ \dots \succ n$.

Insertion sort strategy: for each $k \in \{n-1, \dots, 1\}$,

– totally rank $\{k+1, \dots, n\}$

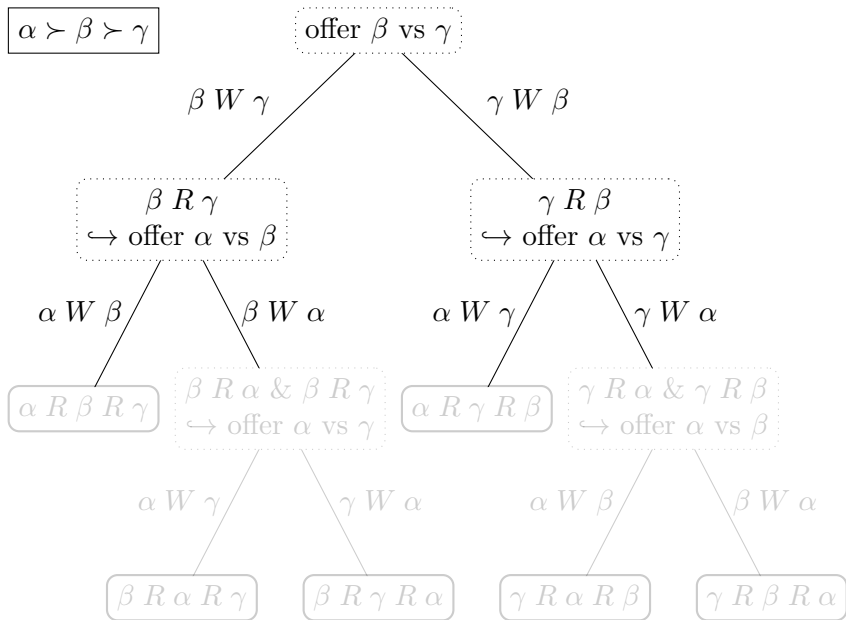
(write $x_{k+1} R \dots R x_n$, where $\{x_{k+1}, \dots, x_n\} \equiv \{k+1, \dots, n\}$)

– ‘insert’ k into $\{k+1, \dots, n\}$:

pit k against the highest-ranked (x_{k+1});

then (if k lost) pit k against the 2nd-highest-ranked (x_{k+2});

...



Insertion sort is regret-free

Theorem 1.

The insertion-sort strategy is efficient, hence regret-free.

Proof of Theorem 1

Fix a W , and let R be the outcome of IS under W .

Fix x, y with $x \succ y$ and $x W y$; we must show that $x R y$.

Enumerate all alternatives \succ -worse than x as $z_1 R \cdots R z_K$.

Note that $z_k = y$ for some $k \leq K$.

By definition of IS,

x is pitted against z_1, z_2, \dots in turn until it wins a vote.

- if x loses against z_1, \dots, z_{k-1} ,
then it is pitted against $z_k = y$ and wins (since $x W y$)
 $\implies x R y$.
- if x wins against z_ℓ for $\ell < k$,
then $x R z_\ell R \cdots R z_k = y$
 $\implies x R y$ (by transitivity of R). ■

What (other) strategies are regret-free?

We've shown that regret-free strategies exist.

What are their characteristics?

Characterisation of outcomes

Recall that W -efficiency \implies W -unimprovability (Lemma 1).

The converse is false:

a W -unimprovable ranking need not be W -efficient.

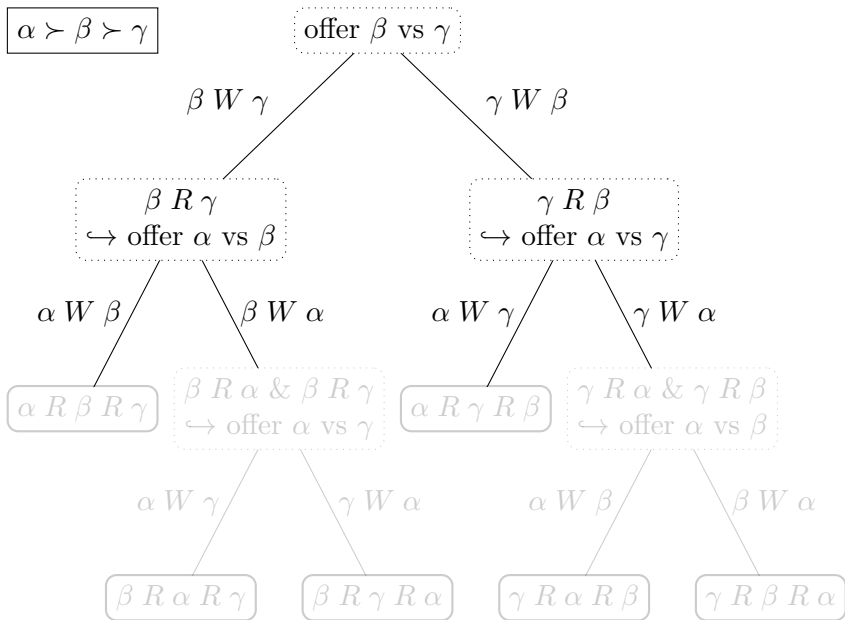
(counter-example: slide 24)

But only efficiency ensures unimprovability robustly across W s:

Theorem 2.

A strategy is regret-free iff it is efficient.

(tightness: slide 25)



Characterisation of behaviour

Theorem 3.

A strategy is regret-free iff
it never misses an opportunity or takes a risk.

(formal definitions: slide 26)

(tightness: slide 27)

History-invariant voting

We have assumed throughout that W is fixed
 \iff voting is (approximately) history-invariant.

Reasonable if voters are unsophisticated or vote expressively.

Not unreasonable if voting is strategic. *(details: slide 28)*

Alexander der fünfft



Gregorius der. viij.



THANKS!

Benedictus der. xiij.



What's special about insertion sort?

For an alternative x , strategy σ and majority will W , write $R^\sigma(W)$ for the outcome of σ under W , and

$$N_x^\sigma(W) := |\{y : x \succ y \text{ and } x R^\sigma(W) y\}|.$$

Definition.

Given $x \in \mathcal{X}$, σ is *better for x* than σ' iff

$$|\{W : N_x^\sigma(W) \geq k\}| \geq |\{W : N_x^{\sigma'}(W) \geq k\}| \quad \forall k \in \{1, \dots, n-1\}.$$

If $\sigma \in \Sigma$ is better for x than each $\sigma' \in \Sigma$, it is *best for x among Σ* .

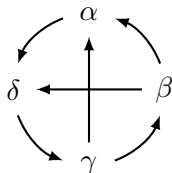
Label the alternatives $\{1, \dots, n\}$ so that $1 \succ \dots \succ n$.

Theorem 4.

A strategy is outcome-equivalent to insertion sort iff
among all strategies, it is best for 1;
among such strategies, it is best for 2; and so on.

Counter-example to the converse of Lemma 1

$\mathcal{X} = \{\alpha, \beta, \gamma, \delta\}$ with $\alpha \succ \beta \succ \gamma \succ \delta$ and



The ranking $\alpha R \delta R \gamma R \beta \dots$

- (– is reachable under W : offer $\{\alpha, \delta\}$, $\{\delta, \gamma\}$, $\{\gamma, \beta\}$.)
- is W -unimprovable,
since no other W -feasible ranking ranks α above β .
(Because there's only one directed path in W from α to β .)
- is not W -efficient, since $\delta R \beta$.

(back to slide 18)

Theorem 2 tightness

The characterisation in Theorem 2 is tight in the following sense:

Proposition 1.

For any W and W -reachable W -efficient ranking R , some regret-free strategy has outcome R under W .

Thus for every majority will W ,

$$\begin{aligned} & \{R : \exists \text{ regret-free strategy with outcome } R \text{ under } W\} \\ &= \{R : R \text{ is } W\text{-reachable and } W\text{-efficient}\} \end{aligned}$$

(\subseteq by Theorem 2, \supseteq by Proposition 1)

(back to slide 18)

Formal definition of errors

A *proto-ranking* is an incomplete ranking: formally, an irreflexive and transitive relation on the set of alternatives.

Definition.

Let R be a non-total proto-ranking, and let $x \succ y$ be unranked.

- (1) Offering $\{x, y\}$ for a vote *misses an opportunity (at R)* iff there is an alternative z s.t. $x \succ z \succ y$ and $y \not R z \not R x$.
- (2) Offering $\{x, y\}$ for a vote *takes a risk (at R)* iff there is an alternative z s.t. either
 - $z \succ y$, $x R z$ and $y \not R z$, or
 - $x \succ z$, $z R y$ and $z \not R x$.

(back to slide 20)

Theorem 3 tightness

Proposition 2.

After any error-free history,
there is a pair that can be offered without committing an error.

Yields tightness:

for any W and any sequence of pairs that is error-free under W ,
some regret-free strategy offers this sequence under W .

(back to slide 20)

Strategic voting

Each voter i has a preference \succ_i over alternatives, and prefers rankings more aligned with \succ_i .

A voter's *strategy* specifies how to vote at each history.

The *sincere strategy*: vote for your favourite. History-invariant!

Outcome of chair [voters] using σ [σ_i, σ_{-i}] denoted $R(\sigma, \sigma_i, \sigma_{-i})$.

Definition.

A strategy σ_i is *dominant* iff for any alternative strategy σ'_i ,

- (\nexists) there exists no profile σ, σ_{-i} such that $R(\sigma', \sigma_i, \sigma_{-i})$ is distinct from, and MAW \succ_i than, $R(\sigma, \sigma_i, \sigma_{-i})$.
- (\exists) there exists a profile σ, σ_{-i} such that $R(\sigma, \sigma_i, \sigma_{-i})$ is distinct from, and MAW \succ_i than, $R(\sigma', \sigma_i, \sigma_{-i})$.

Proposition 4.

The sincere strategy is (uniquely) dominant.

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